Shack-Hartmann wavefront sensor sensitivity loss factor estimation in partial correction regime

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ABSTRACT

In typical adaptive optics applications, the atmospheric residual turbulence affects the wavefront sensor response decreasing its sensitivity. On the other hand, wavefront sensors are generally calibrated in diffraction limited condition, and, so, the interaction matrix sensitivity may differ from the closed loop one. The ratio between the two sensitivities, that we will call the sensitivity loss factor, has to be estimated to retrieve a well-calibrated measurement. The spots size measurement could give a good estimation, but it is limited to systems with spots well sampled and uniform across the pupil. We present an algorithm to estimate the sensitivity loss factor from closed loop data, based on the known parameters of the closed loop transfer functions. Here we preferred for simplicity the Shack-Hartmann WFS, but the algorithm we propose can be extended to other WFSs.

1. INTRODUCTION

In this article we focus on the sensitivity gain of the Wavefront Sensor (WFS), that is the ratio between the calibrated sensitivity and the one obtained during operation. For many applications we need the actual WFS sensitivity to get the correct measurement (very important in open-loop), to proper optimize the temporal controller, and to proper compensate for Non Common Path Aberrations (NCPA). A change on the WFS sensitivity can be caused by a variation of the spot size\textsuperscript{1} (strong effect in a quadcell SH) or a change of dark current and/or sky background. In some cases these effects can be measured and so the sensitivity loss factor can be estimated,\textsuperscript{2–4} otherwise a specific algorithm should be used to estimate this.\textsuperscript{5,6}

In this paper we present an approach to the estimation of this coefficient based on the closed loop Transfer Function (TF) and on the loop data Power Spectral Densities (PSD).

2. CLOSED LOOP TRANSFER FUNCTIONS AND WFS SENSITIVITY

Let us consider a Single Conjugate Adaptive Optics (SCAO) closed loop system as the one shown in Figure 1. Note that we choose a SCAO system for simplicity, however other systems, like multi-conjugate adaptive optics\textsuperscript{7,8} MCAO, can also be considered as well. The command vector $c$, and the incremental command vector $\Delta c$ are defined as:

\begin{equation}
    c = \frac{gH}{1 + gH} t + \frac{gSRCD}{1 + gH} v,
\end{equation}

\begin{equation}
    \Delta c = \frac{WSR}{1 + gH} t + \frac{SR}{1 + gH} v,
\end{equation}

where:

\begin{equation}
    H = WSRCD,
\end{equation}

$W$ is the WFS (optics + detector), $S$ is the slope computer, $R$ is the reconstruction matrix, $D$ is the deformable mirror, $g$ is the integrator gain vector, $C$ is the controller, $t$ is the turbulence and $v$ the measurement noise vector.

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Figure 1. Scheme of a SCAO closed loop system.

Figure 2. Left: RTFs and NTFs of an AO system with a sampling frequency of 1000Hz, 2 frames of delay, an integrator gain of 0.5 and different WFS sensitivity loss factors (1 and 0.7). Right: Ratio between RTFs (or NTFs) shown on the left.
The effect of the sensitivity loss factor $\alpha$ on RTF and NTF is shown in Figure 2, and on command and incremental command in Figure 3.

Note that it is possible to estimate the RTF, as shown in Desenne et al., but, unfortunately, this estimation is not affected by the sensitivity loss factor: in fact, if we compute this TF using Equations 5 and 6, and $z^{-k} = W'$, that is the delay to be added to the commands to be synchronized with the incremental commands, we get:

$$\frac{\Delta c}{\Delta c + cz^{-k}} = \frac{1}{1 + gCDz^{-\gamma}},$$

and so, $\alpha$ has been reduced in the computation.
3. SENSITIVITY ESTIMATION

The sensitivity loss factor, \( \alpha \), estimation is evaluated minimizing the cost function in the frequency range \([\omega_1, \omega_2]\):

\[
J(\alpha) = \sum_{\omega=\omega_1}^{\omega_2} (\text{PSD}\{\Delta c(\omega)\} - \hat{K}(\omega, \alpha))^2 ,
\]

(10)

where:

\[
\hat{K}(\omega, \alpha) = \left( \frac{\alpha W'SR}{1 + gaH^2} \right)^2 \text{PSD}\{t(\omega)\} + \left( \frac{SR}{1 + gaH^2} \right)^2 \text{PSD}\{v(\omega)\} ,
\]

(11)

and the operator:

\[
\text{PSD}\{x(\omega)\} \triangleq \langle |x(\omega)|^2 \rangle
\]

(12)

denotes the temporal Power Spectral Density (PSD) of \( x \).

So, summarizing, we search for the sensitivity loss factor, \( \alpha \), which minimizes the difference between the incremental command PSD and the PSD, computed filtering the inputs PSDs by the \( \alpha \) dependent TFs.

Note that here we suppose to know exactly both the turbulence \( t \) and the measurement noise \( v \). In a real system these data should be estimated before running the sensitivity loss factor estimation.

4. EXAMPLES

In this section we present two examples, showing the effect of a spot size change on sensitivity for a quad-cell SH WFS and the effect of a background variation on a Center of Gravity (CoG) SH WFS. For both examples, we run a numerical simulation of a simple system, with a single sub-aperture SH and with a phase containing only tip-tilt, using the PASSATA software.\(^{10}\)

The parameters of the simulations are:

- WFS FoV = 2.5arcsec;
**5. CONCLUSIONS**

An accurate estimate of the WFS sensitivity is essential to get the correct measurement amplitude in the AO loop. We show in this paper how the WFS sensitivity affects the transfer functions of the closed loop, producing measurable effects on the closed loop and inputs data PSDs, so that PSD analysis can be used to estimate the sensitivity gain. The main limitation of this work is that, in most cases, a direct measurements of the input...
data is not available. This is the reason why further work is needed to find a method to estimate the WFS sensitivity when only closed loop data is available.

On the other hand, a robustness check of the evaluation of the sensitivity gain with respect to the knowledge of the turbulence and with respect to WFS noise estimated, may provide information about the operative range of the method described herein.

REFERENCES


