Response functions for NLTE lines

\[ n_i \propto g_i e^{-E_i/kT} \]

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Main messages:

- We present an accurate and fast method for computing level population responses in NLTE
- This method could speed up the inversion process by an order of magnitude
- Idea: A simple form of Hanle inversion can be easily devised from scalar NLTE inversion
- Anisotropy response function gives us better height span than the one for intensity alone
Response functions to temperature for a prototype 2-level NLTE line. 1000 G magnetic field.
Response functions

\[ R_{q_k} = \frac{dI^+}{dq_k} \]

They tell us how the emergent intensity responds to perturbations of different physical parameters at different depths.

\[
\frac{d\hat{I}}{ds} = -\hat{K}\hat{I} + \hat{j}
\]

Usual approach (e.g. SIR, SPINOR):

- Compute perturbations in absorption/disappearance/emission terms
- Propagate them analytically to the top
- Done!

Problems in NLTE:

- Level populations are not explicitly given
- Dependencies are non-local and non-linear
- So are the responses
- How to proceed?
NLTE vs LTE

Response function to temperature for a prototype 2-level NLTE line. Unpolarized (isotropic) case.
The key is computing the population responses

\[ n_i \propto g_i e^{E_i/kT} \]

Statistical equilibrium instead of Saha-Boltzmann equation!

\[ \frac{dn_i}{dt} = \sum_j n_j R_{ji} - n_i R_{ij} = 0 \]

All the non-locality and non-linearity is here.

\[ \frac{dR_{ij}}{dq_k} = \frac{\partial R_{ij}}{\partial J_{ij}} dJ_{ij} + \frac{\partial R_{ij}}{\partial q_k} \]

All the non-locality and non-linearity is here.

\[ J_{ij} = \int \phi_{ij}(\lambda) d\lambda \int \frac{d\Omega}{4\pi} I(\Omega, \lambda) \]
And now (bear with me):

\[
\frac{dR_{ij}}{dq_k} = \frac{\partial R_{ij}}{\partial J_{ij}} \frac{dJ_{ij}}{dq_k} + \frac{\partial R_{ij}}{\partial q_k}
\]

\[
J_{ij} = \int \phi_{ij}(\lambda) \, d\lambda \int \frac{d\Omega}{4\pi} I(\Omega, \lambda)
\]

\[
I_l = \int_0^\infty \mathcal{O}(\tau) S(\tau) \, d\tau = \Lambda[\chi, j]
\]

\[
\frac{dI_l}{dq_k} = \sum_{l'} \sum_{i'} \left[ \frac{\partial I_l}{\partial \chi_{i'}} \frac{\partial \chi_{i'}}{\partial n_{l'}} + \frac{\partial I_l}{\partial j_{i'}} \frac{\partial j_{i'}}{\partial n_{l'}} \right] \frac{dn_{i'}}{dq_k} + r_{l', i'}
\]

Which leads to a linear system of ND x NL equations:
Final linear system

\[
\begin{pmatrix}
T_{1,1,1,1} + N_{1,1,1,1} \\
. \\
. \\
. \\
. \\
. \\
\end{pmatrix} + \begin{pmatrix}
T_{l,l,i,i} + N_{l,l',i,i'} \\
. \\
. \\
. \\
. \\
. \\
\end{pmatrix}
\begin{pmatrix}
\cdots \\
N_{1,N_D,1,N_L} \\
\end{pmatrix} = \begin{pmatrix}
\cdots \\
\hat{\mathcal{R}} \\
\end{pmatrix} + \begin{pmatrix}
\cdots \\
b_{i,l,k} \\
\end{pmatrix}
\]

Non-local, radiative (a la lambda operator) coupling

Local terms (surviving from SE equation)

Level responses

All perturbations which are **not** due to level perturbations (known stuff)
Does it work?

- A simplified example of CaII 8542 (5 levels, LTE continuum opacity/emissivity, no magnetic field)
- Fast: Same as one NLTE solution
- Accurate and, in the cases considered- robust
Advantages

- Main application are inversion codes.
- Numerical responses to nodes take time of ~ 10 NLTE solutions → we save an order of magnitude
- Coupling matrix does not depend on perturbation in question → we can perturb and compute whatever we want
- Studying responses themselves is interesting and this way we can do it quickly (i.e. for MHD cubes)
Application to scattering line polarization

Hanle diagnostics so far (unresolved observations):

- Compute anisotropy & collisional depolarization
- Fit to get magnetic field.

Milic & Faurobert, 2012

Trujillo Bueno et al, 2004
Application to scattering line polarization

Hanle diagnostics for resolved observations:

- **Invert, in NLTE, intensity profile.**
- Compute anisotropy & collisional depolarization
- Fit to get magnetic field.

Let's try and compute response functions for scattering polarization!

\[
S_Q \approx (1 - \mu^2)^{\frac{3}{8}} \iiint (3\mu'^2 - 1)I(\mu', \lambda)\phi(\lambda)d\lambda d\mu'
\]
Scattering polarization responses

- Strong, scattering dominated line, formed high in the atmosphere.
- Polarization responses more non-local, and more "extended".
- Better diagnostics?
Main messages (again):

- We present an accurate and fast method for computing level population responses in NLTE.
- These can speed up the inversion process by an order of magnitude, and in preliminary testing work well.
- This approach straightforwardly leads to anisotropy responses and thus to the first approximation of the response of scattering polarization.

Questions? Critics? Comments?