Understanding dynamo mechanisms from 3D convection simulations of the Sun

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Solar Activity

differential rotation

turbulent convective motions

alpha effect

pumping

diffusion

no direct measurements
\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B) - \nabla \times \eta J \]

\[ B = \overline{B} + b' \quad u = \overline{U} + u' \]

\[ \frac{\partial \overline{B}}{\partial t} = \nabla \times (\overline{U} \times \overline{B} + u' \times b') - \nabla \times \eta \overline{J} \]

\[ \nabla \times (\overline{U} \times \overline{B}) = (\overline{B} \cdot \nabla)\overline{U} - \overline{B} (\nabla \cdot \overline{U}) - (\overline{U} \cdot \nabla)\overline{B} \]
Electromotive force

\[ \mathcal{E} = a \cdot \overline{B} + b \cdot \nabla \overline{B} + \ldots \]

\[ \mathcal{E}_i = a_{ij} \overline{B}_j + b_{ijk} \partial_j \overline{B}_k + \ldots \]

\[ \mathcal{E} = \alpha \cdot \overline{B} + \gamma \times \overline{B} - \beta \cdot (\nabla \times \overline{B}) - \delta \times (\nabla \times \overline{B}) - \kappa \cdot (\nabla \overline{B})^{(S)} \]
\[ \frac{\partial \overline{B}}{\partial t} = \nabla \times (\overline{u} \times \overline{B} + u' \times B') - \nabla \times \eta \nabla \times \overline{B}, \]

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\[ \frac{\partial B'}{\partial t} = \nabla \times (u' \times \overline{B}^T + \overline{u} \times B' + u' \times B' - u' \times B') - \nabla \times \eta \nabla \times B' \]
The Simulation
Global convective dynamo simulations

\[
\frac{\partial A}{\partial t} = u \times B + \eta \nabla^2 A
\]

\[
\frac{D \ln \rho}{Dt} = -\nabla \cdot u
\]

\[
\frac{Du}{Dt} = g - 2\Omega_0 \times u + \frac{1}{\rho} \left( J \times B - \nabla p + \nabla \cdot 2\nu \rho S \right)
\]

\[
T \frac{Ds}{Dt} = \frac{1}{\rho} \nabla \cdot (K \nabla T + \chi_t \rho T \nabla s) + 2\nu S^2 + \frac{\mu_0 \eta}{\rho} J^2 - \Gamma_{cool}(r),
\]

- high-order finite-difference code
- scales up efficiently to over 60,000 cores
- compressible MHD

https://github.com/pencil-code/pencil-code/

Warnecke et al. 2014, 2016
Results
In Sections 4.1–4.4 we focus on the analysis of the time-
mean density. For a direct comparison we plot the meridio-
al profiles of the diagonal components of the kinetic helicity
with a cos profile. As found by, e.g., Käpylä et al. (2006a) for
moderate rotation. However, beside the negative radial shear
is overall similar to that of Brandenburg & Sokolov (2002; Kowal
et al. 2006) as well as different from ours.

Among the three orthogonal components of the kinetic helicity with a cos profile.

\[ P(\alpha_{ij}) = \frac{(\alpha_{ij}^{es})^2 - (\alpha_{ij}^{ca})^2}{(\alpha_{ij}^{es})^2 + (\alpha_{ij}^{ca})^2}, \]

is around 6 \( \theta \), and is therefore significantly weaker than in the northern hemisphere of Run I and for three different realizations at surface due to radial timescale. This sign reversal with latitude near the surface, but also in deeper layers, where its strength is smaller, see Figs. 1 and 2. The latitudinal dependence of the kinetic helicity distribution as found by, e.g., Käpylä et al. (2006) for moderate rotation. However, beside that, 

is opposite to that near the surface. This sign reversal with latitudes near the surface, but also in deeper layers, where its strength is smaller, see Figs. 1 and 2. The latitudinal dependence of the kinetic helicity distribution as found by, e.g., Käpylä et al. (2006) for moderate rotation. However, beside that

is much weaker is in the middle of the convection zone and below; but the values are still high compared to the other profiles at the bottom right at each panel: overall parity is main.

\[ \alpha \equiv \nabla \cdot \alpha = \mu_0 \epsilon_0 \alpha, \]

are mainly positive in the north and negative in the south, but have a sign reversal in the low-dissipation limit (Pouquet et al. 1976) via similar to that of 

\[ \frac{\partial \alpha}{\partial t} = \mu_0 \epsilon_0 \nabla \times (\nabla \times \alpha) \]

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The dominant effect is strong close to the surface, producing positive (negative) toroidal field at high (mid) latitudes (mostly opposite sign in the mid-latitude regions of strong field production). On one hand, the effect is stronger than or comparable to the high frequency dynamo mode found near the surface by Warnecke et al. (2014) and Käpylä et al. (2016a) for similar runs. The toroidal field generates positive toroidal magnetic field generation via \( \alpha \· B \), which has only one third of the strength as the main toroidal field generators, but shows the opposite sign in the mid-latitude regions of strong field production. However, for these regions there seems to be no clear relation to toroidal field concentrations at this instant of the cycle. However, it exhibits far stronger spatial variations than the contribution of the turbulent di\( m\)o\( n\)y near the surface might explain the measured distribution. 

Directly next to this region, further away from the tangent cylinder, the \( \alpha \) effect at mid latitudes just outside the tangent cylinder, therefore enhancing its negative toroidal field production transport coefficients, see Section 3.1. We note here that a simplified treatment with \( \alpha \· B \) \( r \) shows also contributions at high latitudes in the upper half of the dynamo near the surface might explain the measured distribution. 

The dominant effect at high latitudes in the upper half of the dynamo is locally dominant over the \( \alpha \) effect, while at low latitudes in the upper half of the dynamo near the surface might explain the measured distribution. 

The contributions of the radial and latitudinal derivatives of \( \alpha \) have only one third of the effect and thus do not match up well with the \( \alpha \) \( \times \cdot B \) \( r \) term to \( \eta B \) \( \times \cdot B \) \( r \) at the half of a typical activity cycle with positive (negative) toroidal field contribution of the turbulent dynamo. 

The convection zone it is stronger. There and near the surface positive (negative) toroidal field at high (mid) latitudes (mostly opposite sign to \( \eta \)) from the tangent cylinder, the \( \alpha \) field similar to the \( \eta \). 

One finds that the production terms. The structures at small spatial scale responding to convection near the surface. However, the production terms are likely to be taken as indication of poor scale separation between mean and fluctuating quantities, pointing to the need of scale dependent transport coefficients, see Section 3.1. 

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Turbulent pumping

\[ \mathcal{E} = \alpha \cdot \overline{B} + \gamma \times \overline{B} - \beta \cdot (\nabla \times \overline{B}) - \delta \times (\nabla \times \overline{B}) - \kappa \cdot (\nabla \overline{B})^{(S)} \]

\[ \frac{\partial \overline{B}}{\partial t} = \nabla \times (\overline{u} \times \overline{B} + \overline{u}' \times \overline{B}') - \nabla \times \eta \nabla \times \overline{B}, \]

\[ \partial_t \overline{B}^{\text{pol}} = \nabla \times \left[ \ldots + \left( \gamma^{\text{pol}} + \overline{U}^{\text{pol}} \right) \times \overline{B}^{\text{pol}} \right] \tag{16} \]

\[ \partial_t \overline{B}^{\text{tor}} = \nabla \times \left[ \ldots + \left( \gamma^{\text{pol}} + \overline{U}^{\text{pol}} \right) \times \overline{B}^{\text{tor}} + \left( \gamma^{\text{tor}} + \overline{U}^{\text{tor}} \right) \times \overline{B}^{\text{pol}} \right] \tag{17} \]
Thus satisfying the first condition, we consider their evolution; upward (downward) pumping such that there (low) latitudes its radial component is dominated by the streamwise-lower (upper) convection zone is also significantly enhanced. For the meridional circulations, as shown by the streamlines in Fig. 5, the equatorward component. As a consequence, the whole meridional circulation is reorganized as seen in Fig. 6, where we plot the radial derivatives of the rotation rate 

\[ \partial_t \Omega \approx \gamma \nabla \times B = r \nabla \times \mathbf{B} \].

The azimuthal flow \( \mathbf{U} \) mainly due to its drastic deviations from solenoidal-ness, the effective meridional circulation inside the solar convection zone, the effective azimuthal flow would still be unknown, because one cannot measure \( \partial_t \mathbf{U} \) inside the Sun. Note, that simultaneous effects of differential rotation and toroidal pumping at the tangent cylinder are no longer present: The three meridional flow cells aligned with the rotation inside the solar convection zone were to determine accurately the meridional circulation. However, this is inconsistent with the presented simulations. Even if helioseismology were to determine accurately the meridional circulation inside the Sun, the effective mean azimuthal flow would still be unknown, because one cannot measure \( \partial_t \mathbf{U} \) inside the Sun.

Also, at this location the toroidal turbulent pumping can move patches of poloidal flux. In principle, it may reach the bottom of the convection zone when transported by the meridional flow. The source term \( \partial_t \mathbf{U} \) alone. For \( \mathbf{U} \), we neglect that the effect of differential rotation and toroidal pumping acts on the source term and hence the differential rotation significantly, as shown in Fig. 5, bottom row). However, this is inconsistent with the presented simulations. Even if helioseismology were to determine accurately the meridional circulation inside the solar convection zone, the effective azimuthal flow would still be unknown, because one cannot measure \( \partial_t \mathbf{U} \) inside the Sun.

Note that, while at least \( \mathbf{U} \) is solenoidal, no such condition applies to \( \gamma \) velocities in comparison to that the poloidal and toroidal constituents of \( \mathbf{B} \) are outside the tangent cylinder are no longer present: The three meridional flow cells aligned with the rotation inside the solar convection zone were to determine accurately the meridional circulation. However, this is inconsistent with the presented simulations. Even if helioseismology were to determine accurately the meridional circulation inside the Sun, the effective mean azimuthal flow would still be unknown, because one cannot measure \( \partial_t \mathbf{U} \) inside the Sun.
Magnetic quenching

In Fig. 12, we plot the variations of the diagonal components of $\alpha$ and $\gamma$ (green lines: mean and median, respectively; blue contours: margin). The fast poleward migrating constituent of the magnetic cycle period), owed to the quadratic effect of the mean field onto the velocity fluctuations. In many cases this is best visualized in Fig. 11. The variations of $\alpha_\phi$, shown as 2D histograms of $\langle \alpha_{\phi i} \rangle$, seem to be predominantly quenched by the radial field, while $\alpha_{rr}$ and $\alpha_{\theta\theta}$, along with $\alpha_{r\phi}$, $\gamma_{r\theta}$, $\gamma_{r\phi}$, and the corresponding toroidal and poloidal components, show a complex interplay.

To quantify the variations further, we plot in Fig. 13 their ratio $\gamma_i/\langle \gamma_{ti} \rangle$, $\beta_i/\langle \beta_{ti} \rangle$, and $\alpha_i/\langle \alpha_{ti} \rangle$. The variations have their appearance when averaged over several cycles. For all shown coefficients these are stronger at low than at higher latitudes. Near the surface (ghosts of which are visible in Fig. 12), discuss the fast poleward migrating constituent of the magnetic cycle period), owed to the quadratic effect of the mean field onto the velocity fluctuations. In many cases this is best visualized in Fig. 11. The variations of $\alpha_\phi$, shown as 2D histograms of $\langle \alpha_{\phi i} \rangle$, seem to be predominantly quenched by the radial field, while $\alpha_{rr}$ and $\alpha_{\theta\theta}$, along with $\alpha_{r\phi}$, $\gamma_{r\theta}$, $\gamma_{r\phi}$, and the corresponding toroidal and poloidal components, show a complex interplay.

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\[ \alpha = \langle \alpha \rangle_t + \alpha^Y. \]
\[ \alpha^Y_{ij} = \sqrt{\langle \alpha^Y_{ij}^2 \rangle_t}, \]
Conclusions

• Test-field method is one way to understand dynamo simulations.
• Alpha deviates from helicity expression.
• Complicated mixture of dynamo effects.
• Turbulent pumping changes significantly the eff. flow.
• Quenching does not depends analytical on $B$
• Strong cyclic variations of coefficients