Numerical non-LTE 3D radiative transfer using a multigrid method

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Next generation instruments

SST/CHROMIS

DKIST 4-meter

National Solar Observatory/AURA/NSF
Partial redistribution: Ca II $K_{2v}$

PRD increases computational work with a factor 10 compared to CRD

Sukhorukov & Leenaarts (2016)
Non-LTE radiative transfer

• Intensity depends on 6D parameter space
• Intensity is non-local
• The problem is non-linear
• MALI scales as $O\left( n^{2}_{\text{points}} \right)$ (Rybicki & Hummer, 1991)

SE equilibrium equation

$$n_{i} \sum_{i \neq j}^{n_{l}} P_{ij}(I_{v}) - \sum_{i \neq j}^{n_{l}} n_{j}P_{ji}(I_{v}) = 0$$

Transport equation

$$\frac{dI}{d\tau_{v}(n_{i})} = S_{v}(n_{i}) - I_{v}$$
Radiative transfer with Multigrid

«False convergence» will occur (similar to Lambda iteration)

- O. Steiner (1990) proved that multigrid works with RT problem
- Väth (1994) multigrid requires large model atmospheres in 3D
- P. Fabiani Bendicho et al (1997) MUGA with non-linear multigrid in 1D and 2D

P. Fabiani Bendicho et al (1997)
Model atmosphere

FAL-C

Extracted column from Bifrost
504x504x497

Electron density [cm$^{-3}$]$	imes 10^{10}$

$T$

$n_e$

Temperature [K]

Height [Km]

$T$

$n_e$

Temperature [K]

Height [Km]

(Fontenla et al. 1993)

(Carlsson et al. 2016)
Idea of multigrid

**Atmosphere**

Grid 1

Grid 2

Grid 3

**SE equilibrium equation**

\[ n_i \sum_{i \neq j}^{n_l} P_{ij}(I_v) - \sum_{i \neq j}^{n_l} n_j P_{ji}(I_v) = 0 \]

Grid 1

\[ n_i \sum_{i \neq j}^{n_l} P_{ij}(I_v) - \sum_{i \neq j}^{n_l} n_j P_{ji}(I_v) \neq 0 \]

Grid 2

\[ n_i \sum_{i \neq j}^{n_l} P_{ij}(I_v) - \sum_{i \neq j}^{n_l} n_j P_{ji}(I_v) \neq 0 \]

Grid 3

**Negative populations allowed at the coarse grid!**
Solutions for negative population

Initialize the population with zero-radiation field

Resolve the problem on the coarse grid
Setup: Multi3D

- Non-linear multigrid in Multi3D (*Leenaarts & Carlsson 2009*)
- Multilevel accelerated lambda iteration (MALI)
- 3D short characteristic solver
- Domain decomposition
Setup: Model atoms

three-level Ca II atom
\[ \epsilon = 10^{-4} \]

six-level hydrogen atom
\[ \epsilon = 10^{-8} \]
Setup: Model atmosphere

3D Bifrost snapshot for $t = 3850$ se, 504 $\times$ 504 $\times$ 496 points

(Carlsson et al. 2016)
Result: three-level Ca II atom

Ca II K 3934.78 Å, $\Delta xy = 48$ km, log(I)

.Method | Speed-up
---|---
Multigrid 3-grid | 3.3x
Full multigrid 3-grid | 6x
Result: Six-level hydrogen atom

<table>
<thead>
<tr>
<th>Method</th>
<th>CPU hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>MALI 1-grid</td>
<td>300 000</td>
</tr>
<tr>
<td>Multigrid 3-grid</td>
<td>68 000</td>
</tr>
<tr>
<td>Full-multigrid 3-grid</td>
<td>38 000 (?)</td>
</tr>
</tbody>
</table>

Used 4096 cores
Result: Speed-up in 1D and 3D

- 17x in 3D??
- 11x in 3D??
- 9x in 3D?
- 6x in 3D

Graph showing speed-up $t_{MAII}/t_{MG}$ vs. $\Delta z$ [km].
High-resolution atmosphere: Bifrost 768³

- 32 km horizontal grid spacing
- 13-100 km vertical grid spacing
- Enhanced network
- LTE Hydrogen population

courtesy of M. Carlsson & V. Hansteen
Study of formation properties of Ca II H&K with 3D PRD is undergoing
Conclusions

• Multigrid with MALI works for MHD snapshots
• Handle strongly scattering lines
• Factor \textbf{4-6x} speed-up for a 504x504x496 MHD snapshot
• Higher speed-up expected for future MHD simulations

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