Data assimilation as a tool to better understand the solar magnetism

Laurène Jouve
IRAP- Toulouse

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In collaboration with S. Brun, C.Hung (CEA – Saclay), A. Fournier (IPG – Paris) and O. Talagrand (LMD – Paris)
Open question:
Predicting future solar activity?

- Why not trying to combine models of solar magnetism and observational data?

Actual maximum for cycle 24 in April 2014.
Physics-based predictions: simple mean-field dynamo models

**Dynamo mechanism:** process through which motions of a conducting fluid can permanently regenerate and maintain a magnetic field against its ohmic dissipation

*It consists of the regeneration of both poloidal and toroidal fields*

**Sources of magnetic field**
- Poloidal
- Toroidal
- Toroidal
- Poloidal

**Transport of magnetic field**
- Large-scale flows (meridional circulation)
- Downward pumping by penetrative convection
- Transport from the base of the convection zone to the surface

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The Sun: meridional flow internal profile

- Some dynamo models (Babcock-Leighton flux transport) using 1 single cell per hemisphere produce butterfly diagrams in agreement with observations.

- BUT from observations and simulations, the MC may be multicellular.

- If a complex profile persists for the whole cycle, the effect on the magnetic field may be dramatic.

Jouve & Brun, 2007

BL model: MC with 4 cells per hemisphere.

Butterfly diagram no longer in agreement with observations.
Combining data and models

- Drive models with data:
  - Magnetic observations into dynamo models
    (Dikpati et al. 2006, Choudhuri et al. 2007)
  - Active regions into surface flux-transport
    (Schrijver & DeRosa 2003, Cheung & DeRosa 2012)

- Assimilate data into models:
  - Purpose: « using all available information to determine as accurately as possible the state of the atmospheric or oceanic flow » (Talagrand, 1997)
  - Operational for weather forecasting for decades

Credit: E. Kalnay
Sequential data assimilation

- Analysis step
  \[ x^a_k = x^b_k + W_k(y_k - H_kx^b_k) \]
- Forecast step
  \[ x^{b_{k+1}} = M_k x^a_k \]

- Used recently in a simple \( \alpha\Omega \) dynamo model (Kitiashvili et al., 2008) and on a BL dynamo model to reconstruct the amplitude of the surface MC by assimilating synthetic magnetic data (Dikpati et al., 2014)
Variational data assimilation

- Different analysis step
  Minimize an objective function to get $x^{a_{k-1}}$

\[
J(\xi) = \frac{1}{2} (x^b - \xi)^T [P^b]^{-1} (x^b - \xi) \\
+ \frac{1}{2} \sum_k [y_k - H_k \xi_k]^T R_k^{-1} [y_k - H_k \xi_k]
\]

with $\xi_{k+1} = M_k \xi_k, \ k = 0, \ldots, K-1$

- Forecast step

\[
x^{b_K} = M_k x^{a_{K-1}}
\]

Propagates information both forward and backward in time

- Used recently in models of solar flares (Bélanger et al. 2007, Strugarek & Charbonneau 2014) and on a simple $\alpha\Omega$ dynamo model to reconstruct the $\alpha$–effect (Jouve et al. 2011)
An example of Var. DA in dynamo models: BLFT model in spherical geometry

Hung, Jouve, Brun, Fournier, Talagrand, 2015

\[ \partial_t A_\phi = \frac{n}{\eta_t} \left( \nabla^2 - \frac{1}{\omega^2} \right) A_\phi - \nabla \ddot{v}_p \cdot \nabla (\omega A_\phi) + C_S S(r, \theta, B_\phi), \]

\[ \partial_t B_\phi = \frac{n}{\eta_t} \left( \nabla^2 - \frac{1}{\omega^2} \right) B_\phi + \frac{1}{\omega} \frac{\partial (\omega B_\phi)}{\partial r} \frac{\partial (\eta/\eta_t)}{\partial r} - Re \ddot{v}_p \nabla \left( \frac{B_\phi}{\omega} \right) - Re B_\phi \nabla \ddot{v}_p + C_\Omega \ddot{v}_p \nabla \times (A_\phi \dot{e}_\phi) \cdot \nabla \Omega, \]

Meridional circulation: the main ingredient

\[ \ddot{v}_p = \nabla \times (\psi \dot{e}_\phi) \]

\[ \psi(r, \theta) = -\frac{2(r-r_{mc})^2}{\pi(1-r_{mc})} \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} d_{i,j} \sin \left[ \frac{i\pi(r-r_{mc})}{1-r_{mc}} \right] P_j^1(-\cos \theta) \quad \text{if } r_{mc} \leq r \leq 1 \]

\[ \text{if } r_{bot} \leq r < r_{mc} \]

\[ m=2, n=4 \Rightarrow 8 \text{ coefs } d_{i,j} \]

<table>
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<th>( d_{1,2} )</th>
<th>( d_{2,1} )</th>
<th>( d_{2,2} )</th>
<th>( d_{2,3} )</th>
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</table>
An example of Var. DA in dynamo models:
Twin experiments

- We produce **synthetic observations** with a given MC and input parameters
- We **noise the data** (normal distribution with std=percentage of $\sigma$)
- We choose a **cost function** to be minimized:

  $$
  \mathcal{J}_A = \sum_{i=1}^{N_o} \sum_{j=1}^{N_o} \frac{[A_{\phi}(R_s, \theta_j, t_i) - A_{\phi}^0(R_s, \theta_j, t_i)]^2}{\sigma_{A_{\phi}}^2(R_s, \theta_j)},
  \mathcal{J}_B = \sum_{i=1}^{N_i} \sum_{j=1}^{N_i} \frac{[B_{\phi}(r_c, \theta_j, t_i) - B_{\phi}^0(r_c, \theta_j, t_i)]^2}{\sigma_{B_{\phi}}^2(r_c, \theta_j)}
  $$

- **Initial guess** for the minimization procedure: a 1 cell MC
- **Initial conditions for direct code**: magnetic field produced by this 1 cell model
- We minimize the cost function by **adjusting the control vector** $d_{i,j}$
- The **diagnostic quantities**:

  $$
  \frac{\Delta p}{p} = \sqrt{\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (d_{i,j} - d_{i,j,true})^2}{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{i,j,\text{true}}^2}}, \quad \mathcal{J}/\mathcal{J}_0 \quad \text{and} \quad \mathcal{J}_{\text{norm}} = \frac{1}{\epsilon} \sqrt{\frac{\mathcal{J}}{N}}, \quad \text{when noisy data}
  $$
An example of Var. DA in dynamo models:

Results without noise

1 cell model

4 cell model

Asymmetric model

Number of iterations
**An example of Var. DA in dynamo models:**

Results with noise

- Results for various noise levels, uniform sampling

- Magnetic field recovery for 30% noise for the 4-cell and the asymmetric cases:
  - 4 cell:  **Aphi2**
  - Asymmetric:  **Aphi3**
A more complete model: time-varying meridional flow

- More challenging attempt: recover a MC with time-varying amplitude and profile

- Produces a modulation in the cycle period and amplitude

Hung, Brun, Fournier, Jouve, Talagrand, submitted
A more complete model: analysis step

- Variational DA is performed every year for 40 years where data is available

- A 10% noise is added to the synthetic data (proxy for SSN and surface magnetic field)

- The MC is recovered to an accuracy of more than 90%
- The error on the reconstructed magnetic field is less than 10%
A more complete model: forecast step

- We produce a forecast for the next solar cycles in our model after 40 yrs of assimilation.

- The predictability horizon is of the order of 1 to 2 cycles.
Conclusions and perspectives

- A proof of concept is established: we can make use of DA in solar physics, sequential as well as variational assimilation.

- DA may be used to infer potentially important ingredients of dynamo models: Hung et al. use a polar coordinate model with a time-varying meridional circulation and recover both its amplitude and profile from noised magnetic data.

- The model was used to produce the data (twin experiments), we now wish to move to real observations and actual predictions (for cycle 25?)

- Longer-term:
  - Apply data assimilation techniques to a full spherical 3D MHD models (many get large scale regular magnetic cycles now, e.g. Ghizaru, Brown, Augustson, Gastine, Käpylä, Warnecke, Hotta, Fan,...)