Lunar eclipse theory revisited: Scattered sunlight in both the quiescent and the volcanically perturbed atmosphere

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Abstract

The residual brightness of the shadowed Moon during a lunar eclipse is attributed to unscattered sunlight rays refracted in the Earth’s atmosphere. The classical theory of lunar eclipses is built on the premise that the sunlight scattered by the gases and particles in the atmosphere contributes negligibly to the brightness of the eclipsed Moon. The current work revisits the lunar eclipse theory, extending it to accommodate spectrally resolved observations and addressing the role of scattered sunlight. Predictions of both direct and diffuse sunlight are produced by integrating the radiative transfer equations over the Earth’s disk. The investigation contemplates scenarios of normal aerosol loading as well as conditions representative of the months and years following a major volcanic eruption. It is shown that omitting scattered sunlight is an acceptable approximation for low and moderate aerosol loadings at visible and longer wavelengths. However, towards the ultraviolet, or at times when the atmosphere contains elevated aerosol amounts, the relative significance of direct and diffuse sunlight may reverse. Spectra of the sunlight that reaches the shadowed Moon during the eclipse are presented to illustrate the distinct contributions from both components. It is also shown that lunar eclipse spectra obtained up to 4–5 years after a major volcanic eruption, such as Mt. Pinatubo’s in 1991, will reveal that the stratosphere remains perturbed above background aerosol levels.

1. Introduction

A lunar eclipse occurs when the Moon enters the cone of shadow cast by the sunlit Earth. Lunar eclipses are relatively rare events, and on average only two of them may be contemplated each year [1]. The brightness and color of the Moon vary significantly during an eclipse, especially if the Moon happens to cross the penumbra, or outer shadow, on to the umbra, or inner shadow. The classical lunar eclipse theory is intended to explain the changing aspect of the eclipsed Moon as viewed from an Earth-bound location [2]. The terrestrial atmosphere plays an essential role in the theory. The refraction of sunlight rays in the stratified atmosphere ensures that sunlight reaches the eclipsed Moon throughout the eclipse. Ozone absorption, as well as Rayleigh, aerosol and cloud particle scattering, contribute to the extinction of sunlight and, in turn, to the evolving appearance of the Moon. In the classical theory, the sunlight scattered in the atmosphere by gas molecules and airborne particles is estimated to contribute negligibly to the illumination of the eclipsed Moon.

Numerous works have provided valuable insight into the theory and observation of lunar eclipses [3–11]. Photometry, in both broad and narrow spectral passbands, has...
traditionally been the preferred technique of observation. Thus, the classical theory is often referred to as the photometric theory of lunar eclipses. Past experiments aided to investigate the phase-dependent disk-integrated brightness of the eclipsed Moon, assess the volcanic and meteoritic origins of stratospheric aerosols, or tentatively infer global and vertical distributions of aerosols, ozone and water in the terrestrial atmosphere. The theory of lunar eclipses establishes the connection between the radiative properties of the atmosphere and the extinction of sunlight observed in disk-resolved photometric images of the Moon.

The spectroscopic characterization of the sunlight that reaches the eclipsed Moon provides unique possibilities in the investigation of the terrestrial atmosphere from the ground [3,9,12]. To mention a few, lunar eclipse spectroscopy may help identify trace species with weak absorption strengths, probe the ozone distribution from absorption in its Chappuis band, or infer the nature of airborne aerosols from the wavelength-dependence of the aerosols’ optical opacity. There is some overlap between what spectroscopic observations of lunar eclipses may provide and the data that in-orbit space missions with instruments operating in solar and star occultation modes routinely report [13–15]. Despite its potentialities, only recently has a high signal-to-noise ratio spectrum of the eclipsed Moon been published [16]. The data were obtained during the partial eclipse of 16 August 2008 from the Observatorio del Roque de los Muchachos on the island of La Palma. The technical difficulty of the observation and the need for a large collecting area that ensured acceptable photon statistics may have deterred earlier attempts.

The current paper revisits the lunar eclipse theory in the visible and near-infrared regions of the spectrum. Thermal emission of the atmosphere and surface of the planet are neglected. The work investigates the contributions from both direct and diffuse sunlight. Foreseeably, scattered sunlight may represent a significant fraction of the sunlight that reaches the eclipsed Moon under conditions of high aerosol content in the atmosphere. Explosive volcanic eruptions, capable of injecting substantial amounts of debris and gases into the stratosphere, provide propitious conditions for the enhancement of scattered sunlight. Our formulation is based on the solution to the radiative transfer equation (RTE) and allows for refraction and multiple scattering by gas molecules and airborne particles. Both continuum and molecular band absorption are taken into account. The current work is largely exploratory and therefore meant to investigate the impact of a diverse range of atmospheric conditions on the lunar eclipse spectra. A later paper will address the specific analysis of the recently published lunar eclipse spectrum [16], which was coincidentally obtained during the meteor shower of the Perseids and a few days after the eruption in Alaska of the Kasatochi volcano.

The paper is organized as follows. Section 2 describes the RTE, model atmospheres and optical properties used in the calculations. Section 3 discusses the direct and diffuse irradiance spectra for a suite of model atmospheres broadly representative of quiescent atmospheric conditions. Section 4 furthers the discussion by including aerosol amounts appropriate to the transient state of the atmosphere after a massive volcanic eruption.

2. Formulation of the radiative transfer problem

The classical theory of lunar eclipses is concerned with imaging the Moon from an Earth-bound location. The angular separation $e$ from the axis of the shadow to the point $O$ on the lunar surface being imaged is a key geometrical parameter in the theory. In contrast to the classical approach, our formulation assumes that the observer sits at $O$ on the lunar surface and images the solar disk from there. We refer to the line that joins the center of the Earth with $O$ as the reference axis, and to the angular separation $e$ as the elevation angle of the Sun above the reference axis. The sketch of Fig. 1 introduces the main geometrical parameters in our formulation.

![Fig. 1. Main geometrical parameters in our treatment of the radiative transfer problem of direct and diffuse sunlight in a lunar eclipse. The reference axis joins the observing point $O$ with the Earth’s center.](image-url)
Sunlight reaches the eclipsed Moon from a range of directions. Denoting by \( L(x_0,s) \) the radiance at \( O \) in a direction \( s \) pointing into the Moon, the irradiance, or radiance integrated over solid angle, at that point is
\[
F(x_0) = \int_{\Omega} L(x_0,s)s \cdot n_{x_0} \ d\Omega(s).
\]
Here, \( n_{x_0} \) is the inward surface vector at \( O \) and \( d\Omega(s) \) is the differential solid angle about direction \( s \).

We focus on the specific case that \( n_{x_0} \) is aligned with the reference axis, which enables us to do \( d\Omega(s) = d\mu \ d\phi \), where \( \mu = s \cdot n_{x_0} \) and \( \phi \) is the azimuthal angle around the reference axis. With the changes, the irradiance at \( O \) becomes
\[
F(x_0) = \int_{\Omega} L(x_0,\mu,\phi) \ d(\mu^2/2) \ d\phi. \tag{1}
\]
In the more general situation that \( n_{x_0} \) and the reference axis are not aligned, the right hand side of the expression above should be premultiplied by the cosine of the tilt angle. The correction is only approximate but acceptable for the small angular size of the Earth as viewed from the surface of the Moon.

The radiance \( L(x_0,\mu,\phi) \) is obtained from the solution to the RTE:
\[
s \cdot \nabla L(x,s) = -\gamma(x) L(x,s) + \beta(x) \int_{\Omega} p(x,s,s') L(x,s') \ d\Omega(s'), \tag{2}
\]
which is complemented with boundary conditions at the Sun and Earth. Here, \( \gamma(x) \) is the extinction coefficient, related to the absorption and scattering coefficients, \( \alpha(x) \) and \( \beta(x) \), respectively, through \( \gamma(x) = \alpha(x) + \beta(x) \), and \( p(x,s,s') \) is the scattering phase function for changes \( s' \rightarrow s \) in the light direction. We assume that \( p(x,s,s') = p(x,\phi) \), where \( \cos \phi = s \cdot s' \) and \( \phi \) is the scattering angle. By construction, \( \phi = 0 \) and \( \pi \) correspond exactly to the forward and backward scattering directions, respectively.

The phase function and the solid angle are normalized such that \( \int p \ d\Omega = 1 \) and \( \int d\Omega = 4\pi \).

2.1. The direct component of irradiance

Part of the sunlight photons that enter the atmosphere travel the entire limb unscattered. These photons follow refraction-bent trajectories, some of them connecting the solar disk with the Moon and thereby contributing to the direct illumination of the eclipsed Moon.

The radiance associated with a sunlight ray that reaches \( O \) in the direction \( s(\mu,\phi) \) is obtained by solving the simplified RTE that results from dropping the scattering term from Eq. (2). Upon integration of the simplified RTE, we have for the direct radiance at \( O \):
\[
L_0(x_0;\mu,\phi) = B_{\odot} U(t(\mu)),
\]
where \( t(\mu) \) is the transmittance along the specific ray trajectory, and \( s \) (not to mistake for \( s \)) is the path length. Provided that the atmosphere is symmetric about the reference axis, the transmittance depends on \( \mu \) but not on \( \phi \). \( B_{\odot} \) is the output solar radiance (same units as \( L_0 \)) at the center of the solar disk, and \( U \) is a limb-darkening factor that accounts for variations in the output radiance across the solar disk. Our current implementation handles linear limb-darkening laws of the type \( U = 1 - u_1(1 - \mu_0) \), where \( u_1 \) is a wavelength-dependent parameter and \( \mu_0 \) is the cosine of the ray’s exiting angle from the solar disk. For the sake of simplicity, all the calculations presented herein use \( u_1 = 0 \), which is equivalent to assuming that the Sun radiates isotropically.

In a stratified medium, light rays bend locally in the direction of positive gradients of the index of refraction [17]. Snell’s law is the mathematical formulation of this principle which, for a spherically symmetric medium, states the invariance of \( R_b = n(r) \cos(\gamma(r)) \), on the ray trajectories. In Snell’s invariant, \( n(r) \) and \( \cos(\gamma(r)) \) are the index of refraction and the impact radius at a radial distance \( r \), respectively. \( R_b \) stands for the angle between the local horizon and the ray direction. The impact radius is the shortest radial distance of the unrefracted ray if traced from its local position and direction. The index of refraction is very approximately one at the top of the atmosphere. Thus, it is fair to regard \( R_b \) as the impact radius of the direct sunlight rays at their entry into, or exit from, the atmosphere.

It is convenient to rewrite Snell’s law in differential form for numerical integration [18]. We integrate the differential equations for the refracted sunlight trajectories from \( O \) towards the solar disk by means of a standard fourth-order Runge–Kutta numerical scheme [19]. Using the path length \( s \) as the variable of integration prevents the singularities that flaw the implementation of other approaches. The numerical scheme takes path length integration steps of 10 km. We have confirmed that the quantity \( n(r) \cos(\gamma(r)) \) remains constant on the ray trajectory to an accuracy that can be arbitrarily set by adjusting the integration step. The index of refraction of the atmosphere varies smoothly with wavelength. Thus, it is justified to assume that the trajectories of two rays of wavelengths differing by a few Ångstroms are nearly identical. The optical opacity does, however, vary rapidly with wavelength near molecular absorption lines. Our integration scheme calculates simultaneously, for intervals a few Ångstroms wide, a single ray trajectory and the transmittances at up to 1000 nearby wavelengths. The entire spectral range is covered by sequentially accumulating as many narrow intervals as necessary.

The sketch of Fig. 1 shows a bundle of refracted sunlight ray trajectories. The direct rays traced from \( O \) with \( R_b \) less than a critical value of \( R_p n(R_p) \) end up hitting the surface of the planet [20]. The rays that do cross the entire limb do so following divergent trajectories. The pattern of ray trajectories is symmetric about the reference axis (\( x \) axis in the graph).

In the evaluation of the direct irradiance at \( O \), we distinguish between the contribution from sunlight rays that pass above a certain top-of-the-atmosphere altitude and the contribution from rays that reach below that level. Whereas the former are assumed to follow straight-line trajectories, the refraction-bent trajectories of the latter are calculated by integrating Snell’s law as described above.

For the calculation of the contribution from refracted sunlight rays, we introduce the normalized variables
\[\xi = (\mu^2 - \mu_M^2)/(\mu^2 - \mu_n^2)\] and \[\eta = \phi/\pi.\] Some algebra shows that \[\mu^2_n = 1 - \left(\frac{r_{\text{TOA}}}{d_0}\right)^2\] and \[\mu^2_M = 1 - \left(\frac{R_e}{d_0}\right)^2,\] where \(r_{\text{TOA}} = R_p + h_{\text{TOA}}\), \(R_p\) is the terrestrial (mean) radius, \(h_{\text{TOA}}\) is the altitude of the top of the atmosphere and \(d_0\) is the distance from \(O\) to the Earth’s center. We take 6377.4, 100 and 382655.9 km for \(R_p\), \(h_{\text{TOA}}\) and \(d_0\), respectively. In the new variables, the contribution to the irradiance from the direct sunlight transmitted through the atmosphere is

\[F^*_D(\xi, \eta) = \pi (\mu^2_n - \mu^2_M) \int_0^1 \int_0^1 L_0(\xi, \eta) \, d\xi \, d\eta.\]

The irradiance \(L(\xi, \eta)\) is equal to \(B \cdot U(\xi)\) for ray trajectories traced from \(O\) intersecting the solar disk, and zero otherwise. For the evaluation of the integral, we use a two-dimensional trapezoidal rule. We find that using 280 and 180 equal-size bins in the \(\xi\) and \(\eta\) directions, respectively, gives a good compromise between accuracy and computational cost.

The contribution to the irradiance from unrefracted sunlight rays is non-zero only when \(O\) is located outside the convergent cone of shadow cast by the planet plus atmosphere. The contribution is computed as the difference of the net output solar irradiance at \(O\) and the irradiance blocked by an opaque sphere of radius \(r_{\text{TOA}}\), that is, \(F^*_{\text{D}}(\xi, \eta) = F^*_D(\xi, \eta) - F^*_{\text{blocked}}(\xi, \eta).\) For the linear limb-darkening law referred to above, it is straightforward to show that, to a good approximation:

\[F^*_D(\xi, \eta) = B_O \pi \left( \frac{R_p}{R_e} \right) \frac{2}{3 - u_1} \cos e'\]  \hspace{1cm} (3)

\(R_S\) is the solar radius, and \(l_O\) and \(e'\) are readily grasped from Fig. 1. For the solar irradiance blocked by the sphere of radius \(r_{\text{TOA}}\), we do \(\xi' = (\mu^2 - \mu^2_M)/(1 - \mu^2_M)\) and \(\eta' = \phi/\pi,\) with \(\mu^2_M\) as defined above, and evaluate:

\[F^*_{\text{blocked}}(O) = \pi (1 - \mu^2_M) \int_0^1 \int_0^1 l_{\text{blocked}}(\xi', \eta') \, d\xi' \, d\eta'.\]

taking \(l_{\text{blocked}}(\xi', \eta') = B_O U\) for straight line trajectories traced from \(O\) that do intersect both the \(r_{\text{TOA}}\)—radius sphere and the solar disk, and zero otherwise. A Monte Carlo integration scheme with \(10^6\) realizations is used to evaluate the integral in the \(\xi'\) and \(\eta'\) variables.

Fig. 2 shows the Earth’s disk and both the refracted and unrefracted images of the Sun as viewed from \(O\) for \(e\) values from 0° to 0.7°. The refracted image of the Sun varies as the star rises above the Earth’s limb. For \(O\) well within the umbra, the refracted image of the Sun is an annular ring around the Earth’s terminator. As \(e\) increases, the rim splits up, and for larger solar elevations the refracted image becomes a single extended area over

Fig. 2. View of the Earth, the Earth’s limb and the Sun from \(O\). Angles are measured with respect to the reference axis. Using the top left panel as a reference: (1) unrefracted image of the Sun; (2) Earth’s disk contour; (3) refracted image of the Sun (not to scale); (4) the atmosphere’s 30-km altitude contour (not to scale).
one hemisphere. Table 1 contains information on the boundaries of the extended area for the atmospheric conditions of our standard model atmosphere. The precise shape of the boundary, however, is sensitive to local and temporary changes in the index of refraction of the medium.

Our treatment of the direct radiative transfer problem differs from the treatment in the classical theory that is generally adopted in other works [5–7,9,10]. The classical theory is devised to circumvent the computational burden of integrating Eq. (1) from the solution to the RTE of Eq. (2). For that purpose, it accounts separately for the effects of refraction and focusing (see below) of sunlight rays, and simplifies the tracing of ray trajectories. Our approach is formally more intuitive, relies exclusively on first principles and treats in a unified manner the umbra and the penumbra. The main challenge in our approach is tracing the bundle of sunlight rays that contribute to the illumination of the selected point \(O\) on the lunar surface. The difficulty is easily coped with if an efficient integration scheme for Snell’s law is implemented.

### 2.2. The diffuse component of irradiance

Part of the sunlight photons that enter the atmosphere are scattered moonward by the gas molecules and particles in the medium. Earlier investigations estimated that diffuse sunlight contributes negligibly to the illumination of the eclipsed Moon in the usual atmospheric conditions of low-to-moderate aerosol loading [2,11]. Occasionally, the atmosphere may contain abnormally elevated amounts of efficient forward-scattering particles that alter the bulk radiative properties of the atmosphere. Explosive volcanic eruptions are likely to inject large amounts of debris and CO\(_2\), H\(_2\)O and SO\(_2\) gases into the troposphere and lower stratosphere. Meteor showers are also known to perturb episodically the particle content of the atmosphere. To the best of our knowledge, a rigorous study with up-to-date data on the content and scattering properties of tropospheric and stratospheric particles has never been made.

Singly scattered sunlight photons reach the eclipsed Moon from virtually anywhere in the limb of the Earth. Besides, sunlight photons may undergo one or more scattering collisions prior to exiting the atmosphere. The occurrence of multiple scattering collisions in a non-trivial geometry makes it difficult to estimate the diffuse irradiance in a lunar eclipse. To cope with this difficulty, we use a reverse multiple-scattering Monte Carlo scheme (MC scheme), which solves the full RTE of Eq. (2) in a spherical shell atmosphere. The unattenuated solar irradiance at the Earth’s orbital distance, \(a_0\), of magnitude \(B_0 \pi (R_0/a_0)^2((3-\mu_1)/3)\delta(\Omega-\Omega_0)\) is introduced in the RTE as a boundary condition in the solar direction. The surface albedo at the planet is assumed to be zero. Obviously, \(\pi (R_0/a_0)^2\) is also the solid angle subtended by the solar disk from the Earth. For \(a_0 = 1.49 \times 10^8\) km, \(\pi (R_0/a_0)^2 \approx 6.8 \times 10^{-5}\). As shown below, the diffuse irradiance at the Moon is largely the result of sunlight photons scattered above the tropopause. Refraction at the corresponding altitudes is of minor importance and is omitted from our calculations of diffuse sunlight. The MC scheme may optionally account for thermal emission, polarization and surface reflection. The scheme has been thoroughly tested against similar approaches [21,22]. As a check, the MC scheme compares the output single-scattering radiances with those obtained from the direct integration of the RTE along the line of sight. The error in the solution produced by a MC scheme is statistical and can be reduced if a sufficient number of realizations is executed. The description and validation of the scheme will be published elsewhere.

As intended originally, the MC scheme produces radiances. For estimating the diffuse irradiance at \(O\), the MC scheme is nested in an implementation of Eq. (1). Because the largest contribution to the diffuse component originates from single-scattering events, Eq. (1) is rewritten in terms of the normalized \(\zeta\) and \(\eta\) variables defined above. The resulting expression for the diffuse irradiance at \(O\) is analogous to that for \(F_d < \text{TOA}(x_0)\), that is:

\[
F_d(x_0) = \pi(\mu_0^2-\mu_0^4)\int_0^1 \int_0^1 L_d(\zeta, \eta) \; d\zeta \; d\eta.
\]

Finally, the diffuse irradiance is estimated from individual realizations of \(L_d(\zeta, \eta)\) produced by the MC scheme. For each realization, a direction is randomly picked from a uniform distribution of \(\zeta\) and \(\eta\). A total of \(10^4\) realizations per spectral bin yield irradiances with relative accuracies of about \(10^{-2}\).

The polarization of diffuse sunlight is negligible for the small scattering angles that occur in single-scattering events in a lunar eclipse. Thus, we ignore the Stokes parameters other than the radiance from our treatment.

### 2.3. Spectral grid, model atmospheres and optical properties

Our treatment of the radiative transfer problem assumes that the wavelength of sunlight photons is not altered in the interaction of the photons with the atmosphere. The simplification precludes Raman scattering, which produces the fitting-in of lines in the solar Fraunhofer spectrum [23]. It does, however, enable us to split the spectral problem into a sequence of monochromatic problems, each at a separate spectral bin. The radiative

**Table 1**

<table>
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<tr>
<th>(deg)</th>
<th>(r^L_0-R_p) (km)</th>
<th>(r^U_0-R_p) (km)</th>
<th>(r^L_0-R_p) (km)</th>
<th>(r^U_0-R_p) (km)</th>
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properties of the atmosphere are prescribed on a grid of contiguous spectral bins. The grid is unevenly spaced, and designed to resolve the rapid variations in the radiative properties near molecular absorption lines. Each absorption line is sampled with no less than three bins per Doppler width near the band center. Our grid for full spectral calculations from 0.35 to 2.5 μm comprise about 2 million bins. Although the manuscript systematically refers to wavelengths, the calculations are carried out in wavenumbers.

In a first part of our largely exploratory work, we contemplate seven model atmospheres. These are meant to represent reference conditions for: (I) a gas atmosphere without O₃ or O₂−X absorption; (II) atmosphere I + O₃ and O₂−X absorption; (III) atmosphere II + aerosols; (IV)−(VII) atmosphere III + thick clouds with tops at 2, 4, 6 and 8 km, respectively. We borrow from FSCATM the subarctic summer model atmosphere profiles up to 100 km for temperature and concentrations of H₂O, CO₂, O₃, N₂O, CO, CH₄, O₂ and N₂ [24]. The radiative properties are prescribed at 33 concentric layers, the spacing between layers being 1 km near the bottom of the atmosphere and 5 km or more above 25 km. As reference aerosol profile, we utilize the January 1999 data published in a 1984–1999 climatology of stratospheric aerosols [25,26]. The selected reference aerosol profile is representative of background aerosol amounts in the quiescent atmosphere. Quiescent conditions refer to the state of the atmosphere long enough after episodic events that might have perturbed the equilibrium, or background, content of particles in the atmosphere. We assume an aerosol albedo of one. The extinction profiles at the wavenumbers reported in the climatology are exponentially extrapolated to higher altitudes. Towards the bottom, it is assumed that the extinction is altitude-independent below 14 km. The extinction coefficients are linearly interpolated between the wavelengths of the measurements. For the extrapolation longwards of 1.02 μm, we use $\beta_{aerosol} \sim \lambda^{-x}$ and estimate the Ångstrom coefficient $x$ from the 0.525 and 1.02 μm data.

In a later part of the discussion, we investigate the eclipse spectrum for abnormally elevated amounts of airborne aerosols. Volcanic eruptions of various intensities occur on a nearly annual basis. Reaction of SO₂ released in the eruption with atmospheric OH leads to sulfur compounds, which trigger the formation and further growth of aerosols. Volcanogenic aerosols undergo multiple physical and chemical transformations, with time scales ranging from hours to years, and take part in the global circulation of the atmosphere. In most volcanic events, the products of the eruption remain confined to the troposphere. In the larger eruptions, however, they are likely to reach the stratosphere either directly or after meridional circulation and subsequent tropical lifting [27]. Once in the stratosphere, it takes years before the atmosphere returns to its pre-eruption aerosol levels. In the last 50 years, at least six volcanoes, namely Agung (1963), Fuego (1974), Mt. St. Helens (1980), El Chichón (1982), Pinatubo (1991) and Cerro Hudson (1991), have perturbed the aerosol levels in the stratosphere at hemispheric and even global scales.

For the exploration of the effect of volcanogenic aerosols, we take the extinction profiles from the January 1985, 1991, July 1991, and January 1992, 1993 and 1995 panels in Fig. 1 of the aforementioned climatology. The profiles are global averages for northern hemisphere latitudes of 15–20°. In January 1985, the atmosphere was still recovering from the eruption of El Chichón, and the aerosol loading was unusually high. Little trace of the eruption remained in the stratosphere by January 1991, and the extinction profile at the time may be considered close to background levels. The explosion of Mt. Pinatubo in June 1991 is the largest volcanic event recorded in the 20th century. Its impact on the global extinction properties of the atmosphere is apparent as early as July 1991. Mt. Pinatubo released about 30 Tg of SO₂ gas. The products of the transformation of SO₂ into aerosols altered the stratospheric nadir-integrated opacity more than tenfold in the months following the eruption [27]. The size distribution of airborne aerosols remained perturbed for up to 5 years. The January 1992, 1993 and 1995 data show different stages in the recovery of the atmosphere after the eruption. By January 1999, the stratosphere can be assumed to have returned to background levels of aerosol.

Besides extinction profiles, the climatology reports effective radii and standard deviations, $r_{\text{eff}}$ and $\sigma_g$, for the unimodal log-normal size distributions of aerosols assumed in the retrievals. For each of the selected months, we have estimated vertically averaged values of $r_{\text{eff}}$ and $\sigma_g$ from Figs. 1 and 10 of the climatology. The estimated values are listed in Table 2. Mie theory is used to calculate the respective vertically averaged scattering phase functions [28,29]. For simplicity, the scattering phase function for each set of $r_{\text{eff}}$ and $\sigma_g$ parameters is calculated at one single wavelength of 0.5 μm. The assumption underestimates slightly the diffuse sunlight at the shorter wavelengths while overestimating slightly its impact at the longer wavelengths.

Positions, intensities and lineshape parameters for molecular absorption bands are borrowed from HITRAN2008 [30]. Approximate Voigt lineshapes are used and truncated

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<th>$r_{\text{eff}}$ (μm)</th>
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<th>$r_g$ (μm)</th>
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500 Doppler widths away from the center of the line [31]. Ozone absorption cross sections at five temperatures between 203 and 293 K are taken from the GEISA compilation and linearly interpolated at intermediate temperatures [32]. Cross sections for the O2–X collisional complex, where X may be either O2 or N2, are implemented for O2–X bands from 343 nm to 1.27 μm [33,34]. Rayleigh scattering cross sections for the more abundant molecules are calculated from the electric-dipole formula:

$$\sigma_{\text{Rayl}} = \frac{32\pi^3\nu^4}{3\nu_0^2}(n_0-1)^2F_k,$$

which includes the King depolarization factor $F_k$ [35]. $n_0$ and $n_\nu$ are the Loschmidt number and the refractive index at one atmosphere and 273.15 K, respectively. The King factor and refractive index for N2, O2, H2O, CO2 and Ar are gathered from a number of sources [35–37]. $F_k=1$ is adopted for water vapor. The refractive index of the background atmosphere for use in Snell’s law is estimated from the weighted summation of the density-scaled refractivities of the main gas constituents [35].

3. Lunar eclipse spectra in the quiescent atmosphere

This first part of the analysis discusses the irradiance spectra calculated for the suite of model atmospheres I–VII. The spectra are normalized to the net output solar irradiance at O, Eq. (3), and degraded with a Gaussian function to local resolving powers of 1000.

3.1. The direct component

The solid curves in Fig. 3 are direct irradiance spectra for model atmosphere II and solar elevations less than or just above the umbra/penumbra edge. The calculation of the dotted curves assumes that the sunlight rays travel unattenuated through the atmosphere. The dotted curves are interpolatable as the apparent angular size of the solar disk as viewed from O relative to its size from outside the eclipse. The size of the solar disk viewed from within the umbra is dictated by refraction and is therefore nearly wavelength-independent in the region of the spectrum investigated. It does, however, vary non-monotonically with the solar elevation. The apparent angular size exhibits a relative maximum when the Sun is imaged from the center of the umbra, and an absolute maximum when is viewed from outside the shadow. The relative maximum for $e=0$ results from the focusing of sunlight rays into the axis of the umbra.

The dashed curves are calculated with Rayleigh extinction as the sole mechanism of sunlight attenuation in the atmosphere. Rayleigh extinction traces the overall appearance of the solid curve spectra. The comparison of the solid curves and the dashed curves makes explicit the absorption in the Chappuis band of ozone and the signature of a few molecular bands of O2, H2O, CO2, CH4 and the O2–X collisional complex at specific wavelengths. For O near the center of the umbra, the refracted sunlight rays are forced to travel longer distances and pass through the denser atmosphere. This explains why the molecular absorption bands appear deeper for small values of $e$.

Interspersed with strong absorption bands, there are a few nearly transparent spectral windows. The refractive focusing of sunlight rays is clearly demonstrated at the windows longwards of ~850 nm. At shorter wavelengths, Rayleigh extinction and ozone absorption partially cancel the effect. For $e \sim 0.6$ and more, the sunlight rays that form the solar image at O miss the bulk of spectroscopically active gases in the lower atmosphere. As O approaches the penumbra/out-of-eclipse edge, the spectrum loses much of its structure.

We have produced spectra such as those on Fig. 3 for all of the I–VII model atmospheres (not shown). Rather than proceed with the discussion of the spectra, we focus on their interpretation at a few wavelengths of 364, 442, 540, 600, 650 and 850 nm. The selected wavelengths avoid strong molecular absorption bands. The wavelength of 600 nm purposefully probes the absorption peak in the Chappuis band of ozone. Fig. 4 shows the light curves of irradiance against solar elevation for all six wavelengths. The vertical lines refer to the umbra/penumbra and penumbra/out-of-eclipse edges at 0.69 and 1.22$^\circ$, respectively. Two recent works have presented light curves spectrally integrated from 400 to 700 nm that in general resemble the curves of our Fig. 4 [3,11]. Note, however, the significant drop in the irradiance in that wavelength interval.

The $\sim \lambda^{-4}$ behavior of the Rayleigh extinction cross section causes the preferential attenuation of the shorter wavelengths that is seen in the light curves for model atmosphere I. Our irradiance for $e=0$ at 600 nm, 3.1 x $10^{-4}$, is roughly consistent with the irradiance averaged from 400 to 700 nm, $3.8 \times 10^{-4}$, quoted in a recent work [11]. Refractive focusing is apparent at the longer wavelengths, but overpowered by Rayleigh extinction at the shorter ones. Model atmosphere II has been partly discussed earlier. It takes I as a basis but includes ~360 Dobson Units (1 Dobson Unit amounts to a nadir-integrated column of $2.69 \times 10^{16}$ cm$^{-2}$) of ozone distributed over a vertical profile that peaks near 20 km. In the spectral range investigated, ozone absorbs in the Chappuis band from 400 to 850 nm. The light curves for II are deeper than those for I at 540, 600 and 650 nm, but are nearly undistinguishable at the other wavelengths. Enhanced absorption at the peak of the Chappuis band causes the crossing of the 600-nm light curve with those for the 540 and 442 nm wavelengths. At the solar elevations for which the light curves cross each other, the sunlight rays graze nearly horizontally the peak of the ozone layer.

Model atmosphere III incorporates the January 1999 aerosol extinction profile. The profile is representative of background levels of aerosols. The nadir-integrated optical opacity of the January 1999 aerosol profile at 500 nm is 0.012 or 0.003 for the atmosphere above 22 km. The aerosols are moderately more absorptive at the shorter wavelengths than at the longer ones. At the center of the umbra, the 540 nm light curve for model atmosphere III drops by about 1.6 with respect to model atmosphere II. The drop is somewhat larger at shorter wavelengths.

Clouds occur in a broad range of altitudes and extend over large and disconnected areas of the globe. Their
optical properties depend on the nature, size distribution and shape of their droplet and crystal constituents. For model atmospheres IV–VII, we consider optically thick clouds capping at 2, 4, 6 and 8 km. Our approach is tantamount to setting an opaque barrier that masks the atmospheric features originating at and below the cloud top. The light curves for model atmospheres IV and V show that low clouds obscure moderately the eclipsed Moon. The obscuration is less evident at the shorter wavelengths because Rayleigh extinction acts naturally as an optical barrier in the cloudless atmosphere. High clouds may entirely block the sunlight rays, as indeed occurs for small solar elevations in model atmospheres VI–VII.

It is worth recalling that we are assuming that the terrestrial atmosphere is symmetric about the reference

Fig. 3. Solid curves: Irradiance spectra for solar elevations between 0° and 0.7°. The spectra are calculated at high spectral resolution, normalized to the irradiance of the unobstructed Sun, and finally convolved with a Gaussian function to a resolving power of 1000. The calculations use model atmosphere II, that includes molecular absorption bands and continuum absorption by ozone and oxygen collisional complexes. Dashed curves: Irradiance for an atmosphere with Rayleigh extinction as the sole mechanism for sunlight extinction. Dotted curves: Irradiance for a fully transparent atmosphere. Because the index of refraction is weakly dependent on wavelength, the dotted curves are nearly color-independent. The solid curves are representative of conditions within the umbra and up to the umbra/penumbra edge, which occurs for $e=0.69°$. Please cite this article as: García Muñoz A, Pallé E. Lunar eclipse theory revisited: Scattered sunlight in both the quiescent and the volcanically perturbed atmosphere. JQSRT (2011), doi:10.1016/j.jqsrt.2011.03.017
axis. The simplification is not critical for $e$ values larger than $\sim 0.3\,$. Far enough from the umbra center, the local atmosphere, rather than the global atmosphere, dictates the fate of sunlight rays that pass through the Earth’s limb. For lower solar elevations, however, the image of the solar disk is formed over an extended region which possibly encircles the disk of the planet. A lunar spectrum measured for $\sim 0.3\,$ or less averages the atmospheric conditions over most of the terrestrial terminator. Indeed, disk-resolved images of the eclipsed Moon show that the asymmetry of the real atmosphere displaces the darkest point in the shadow from the center of the umbra towards the umbra/penumbra edge [3].

### 3.2. The diffuse component

Singly scattered sunlight photons that reach the eclipsed Moon are deflected by less than $2\,$° in the atmosphere. Thus, the behavior of the scattering phase function, $p(\phi)$, near the forward scattering direction, $\phi = 0$, becomes critical in the treatment of the diffuse component.

In the Rayleigh limit, which applies to gas molecules and particles of size much smaller than the wavelength of the incident light, the phase function $p(\phi)$ is a smooth, symmetric, i.e. $p(\phi) = p(\pi - \phi)$, function of $\phi$ that does not strongly favor scattering in the forward or backward directions. Mie theory applies to mid-sized particles, and provides the scattering phase function and other optical properties needed in the radiative transfer calculations. Mie theory assumes spherical, isotropic, homogeneous particles. Given the uncertainties associated with the nature, composition and size distribution of atmospheric aerosols in most practical situations, Mie theory is frequently used beyond its a priori limits. It is well documented that the effective size parameter, $x_{\text{eff}} = 2\pi r_{\text{eff}} / \lambda$, where $r_{\text{eff}}$ is a certain measure of the aerosol size distribution, is critical to explain the behavior of $p(\phi)$ near $\phi = 0$ [38]. Typically, large values of $x_{\text{eff}}$ lead to phase functions that concentrate more and more of the scattered light into a narrow interval of scattering angles near the forward direction. This conclusion is confirmed by geometrical optics, which takes over Mie theory in the limit $x_{\text{eff}} \gg 1$. In geometrical optics, the strong peak in the forward direction is caused by light diffracted past the scattering particle. Fig. 5 shows the phase functions calculated from Mie theory for the aerosol size distributions listed in Table 2. The January 1992 aerosols, about 6–7 months after the eruption of Mt. Pinatubo, forward-scatter the incident sunlight four times more efficiently than the background aerosols of January 1991 and 1999. In January 1992 the size distribution of aerosols was biased towards large droplet sizes after months of coagulation and growth. At the other end of the lifecycle of aerosols, by January 1991 and 1999 the atmosphere had already precipitated the larger-sized aerosols formed from the eruptions of El Chichón and Mt. Pinatubo, respectively.

For our diffuse irradiance calculations, we focus on wavelengths shorter than $1\,\mu m$ and omit the molecular band structure. There being only a few strong molecular absorption bands shortwards of $800\,\text{nm}$, the simplification results acceptable. The diffuse irradiance is nearly independent of $e$ for the small solar elevations within the eclipse shadow. The calculations of the diffuse component presented here are specific to $e = 0$. Fig. 6 shows diffuse irradiance spectra (labeled with a d prefix) for model atmospheres I–III and VII. We note that our diffuse irradiances are roughly consistent with the values of $2 \sim 8 \times 10^{-6}$ quoted in an earlier work for the case of an...
isotropically scattering gas-only atmosphere [2]. Comparison of dI and dII highlights the signature of the Chappuis band of ozone in the latter. The dIII curve shows that a moderate amount of aerosols suffices to enhance the diffuse irradiance by one order of magnitude. In the instances that we have explored, the contribution to the diffuse irradiance from sunlight photons scattered more than once is typically of a few percent. It is shown below that the altitudes below 10–15 km contribute in a moderate manner to the diffuse irradiance. As a consequence, the presence of tropospheric clouds has a relatively minor impact on the diffuse sunlight irradiance, especially at the shorter wavelengths. Comparison of dIII and dVII in Fig. 6 gives evidence of this statement.

Fig. 6 includes also direct irradiance spectra (D prefix) for \( e=0.3 \) and model atmospheres I–III and VII, and for \( e=0, 0.1, \) and \( 0.2 \) and model atmosphere III. The direct and diffuse components become comparable at \( \sim 400 \text{ nm} \) for \( e=0.3 \) and at longer wavelengths for smaller solar elevations.

### 3.3. The stratospheric contribution to the irradiance components

It is worth breaking down by altitude the radiance contributions to the irradiance. For simplicity, we focus on the symmetric configuration \( e=0 \). The thin curves in Fig. 7 show the direct and diffuse radiances, \( L_d(r_b)/B_0 \) and \( L_d(r_b)/B_0 \), respectively, against the impact radius, \( r_b \), for model atmosphere III at selected wavelengths. For scattered sunlight photons, \( r_b \) is defined at the exit of the photon from the atmosphere towards \( O \), as shown in the sketch of Fig. 1. According to Table 1, only impact radii from \( r_b-R_p=1.8 \) to 5.7 km contribute to the direct irradiance for \( e=0 \). For larger impact radii \( r_b \), the direct ray trajectories miss the solar disk, whereas for smaller impact radii the trajectories hit the surface of the planet. Only for the shorter wavelengths does the diffuse irradiance dominate over the direct component. The conclusion could have been anticipated from the comparison of curves DIII \( (e=0) \) and dIII in Fig. 6. \( L_d(r_b)/B_0 \) peaks in the stratosphere at ultraviolet and visible wavelengths. The peak shifts towards the troposphere for longer wavelengths. The thick curves in Fig. 6 show radiance profiles for a model atmosphere that incorporates the January 1992 extinction profile. Larger opacities in the
troposphere and lower stratosphere shift upwards the peak in the diffuse radiance profile. The impact on the irradiance is, as shown below, only moderate.

4. Lunar eclipse spectra in the volcanically perturbed atmosphere

Pre-eruption and post-eruption size distributions of aerosols differ in that the latter typically peak at larger sizes and exhibit broader wings. The enrichment of the atmosphere with large-size particles enhances the atmosphere's efficiency for forward scattering of the incident sunlight. In this section, we compare direct and diffuse irradiances calculated for atmospheric conditions that include amounts of airborne aerosols above the background level. The exercise provides insight into the eclipses that took place between 1985 and 1999 and is expected to guide in the interpretation of future observations.

Each panel in Fig. 8 shows three spectra. One spectrum is for the direct irradiance at \( e = 0.3 \) in a cloudless atmosphere. Another spectrum is for the direct irradiance at the same solar elevation in an atmosphere containing optically thick clouds with tops at 8 km. The last spectrum represents the diffuse irradiance. Molecular absorption bands are omitted. It is apparent that the magnitude of the direct irradiance is strongly dependent on the limb-integrated opacity. The direct irradiances are lowest in July 1991 and January 1992, less than one year after the eruption of Mt. Pinatubo. In July 1991, the direct irradiance is about 3 orders of magnitude lower than that predicted for background aerosol conditions. In January 1991, the drop is even larger. Beginning in 1992, the magnitude of the direct illumination starts its recovery to values expected for the quiescent atmosphere.

The diffuse irradiance spectra in Fig. 8 demonstrate that the diffuse irradiance is significantly less sensitive to the amount of airborne aerosols than the direct component. The diffuse sunlight component varies by only a few for the range of conditions investigated. For further testing this conclusion, we have taken the January 1999 aerosol profile and re-scaled the extinction profile by factors of up to 50. Additional calculations (not shown) indicated that the radiance peak shifts to higher altitudes as the opacity increases. Interestingly, the irradiances at visible and shorter wavelengths remained within a factor of two of their original values. For comparison purposes, we have added to Fig. 7 the profiles of both direct and diffuse radiances obtained for the January 1992 aerosols. The differences in the diffuse irradiances between panels in Fig. 8 are mainly attributable to the forward-scattering efficiency of each specific aerosol size distribution.

There are obvious differences in the appearance of the overall, i.e. direct plus diffuse, spectra predicted for the 1985–1999 period. In conditions of low and moderate aerosol loading, the baseline of the spectrum at visible and near-infrared wavelengths is dictated by direct sunlight and drops rapidly towards the shorter wavelengths. Diffuse sunlight becomes dominant in the near ultraviolet, at \( \sim 400 \text{ nm} \), which translates into a roughly flat net irradiance spectrum shortwards of there. However, for conditions of elevated aerosol loading characteristic of the volcanically perturbed atmosphere, diffuse sunlight becomes competitive at visible and even longer wavelengths. The reason for this is twofold. On the one hand the direct component drops, possibly by orders of magnitude, with respect to the background case, and on the other hand the diffuse component may become moderately enhanced. Again, the situations for July 1991 and January 1992 are quite unique as in those two instances diffuse sunlight dominates over most or all of the
visible and near infrared and the overall spectrum appears rather flat. Thus, the shape of the overall spectrum, especially at the shorter wavelengths, is indicative of the aerosol loading of the atmosphere. Fig. 8 demonstrates that in January 1985, more than 3 years after the eruption of Mt. Pinatubo, the atmosphere remained perturbed by volcanogenic aerosols.

5. Conclusions

The paper revisits the theory of lunar eclipses. Calculations of the direct component of the Moon’s brightness due to refracted sunlight rays were produced, and estimates of the sunlight deflected in scattering collisions with the atmospheric medium presented. The calculations explore a broad range of atmospheric conditions, including various stages in the recovery of the atmosphere after a large volcanic eruption. Our effort benefits significantly from the progress made in the last few decades in the characterization of the stratosphere and stratospheric aerosols.

The image at the Moon of the solar disk is formed over an extended region of the Earth’s limb. The region boundaries vary during the eclipse and may include one or both hemispheres. The changing geometry leads to direct irradiance spectra of evolving appearance. The depth of molecular bands in the spectra varies notably during the eclipse. The strongest bands occur when the observer is located well within the umbra and sunlight photons are forced to travel through the dense atmosphere. In the penumbra, the lunar eclipse spectrum is comparatively much less structured than in the umbra.

The entire terminator scatters sunlight in the direction of the eclipsed Moon. In this respect the direct and diffuse components differ notably. We show that an atmosphere with a background level of aerosols scatters about tenfold more efficiently than a virtual aerosol-free atmosphere. However, increased amounts of aerosols above the background level do not necessarily result in enhanced scattered sunlight. The magnitude of the diffuse component is very sensitive to the effective radius of the size distribution of aerosols. Size distributions that peak at large particle sizes tend to concentrate more of the scattered light near the forward direction, thereby enhancing the amount of moonward-scattered sunlight.

The light that reaches the eclipsed Moon is a blend of direct and diffuse sunlight. The contrast of the two components depends principally on the occurrence of clouds, the content of aerosols and the viewing geometry. In an atmosphere with a background level of aerosols, the two components become comparable at \( \epsilon = 0.31 \) for solar elevations less than 0.3°. Diffuse sunlight contributed effectively to the Moon’s visible brightness in the 3–4 years following the eruption of Mt. Pinatubo. Months after the eruption, it was the largest contributor.

Acknowledgment

The calculation of the scattering phase functions is based on the Mie-theory code developed by Mishchenko [28,29].

References


