On the Origin of the Angular Momentum of Galaxies: Cosmological Tidal Torques supplemented by the Coriolis Force

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ABSTRACT

We present here a theoretical model which can at least contribute to the observed relation between the specific angular momenta of galaxies and their masses. This study offers prima facie evidence that the origin of angular momentum of galaxies could be somewhat more complex than previously proposed. The most recent observations point to a scenario in which, after recombination, matter was organized around bubbles (commonly termed voids), which acquired rotation by tidal torque interaction. Subsequently a combination of the effects of the gravitational collapse of gas in protoplanets and the Coriolis force due to the rotation of the voids could produce the rotation of spiral galaxies. Thereafter the tidal interaction between the objects populating the quasi-spherical voids, in which the galaxies far away from the rotation axes (populating the sheet forming the surface of a void) interact with higher probability with others similarly situated in a neighbouring void, offers a mechanism for transforming some of the galaxies into ellipticals, breaking their spin and yielding galaxies with low net angular momentum, as observed. This model gives an explanation for those observations which suggest a tendency of galactic spins to align along the radius vectors pointing towards the centres of the voids for ellipticals/SO and parallel to filaments and sheets for the spirals. Furthermore, while in simple Tidal Torque Theory the angular momentum supplied to galaxies diminishes drastically with the cosmic expansion, in our approximation for which the Coriolis force acts in addition to tidal torques, the Coriolis force due to void rotation ensures almost continuous angular momentum supply.

Key words: (cosmology:) large-scale structure of Universe

1 INTRODUCTION

Disc-dominated galaxies are the morphologically dominant class of galaxies in the present-day universe (Bamford et al. (2009)). As these galaxies are rotationally supported, it is of considerable importance to understand how they acquire their angular momentum. In the standard scenario of cosmological galaxy formation, galactic halos acquire their angular momentum via tidal torques (Peebles (1969)) in the linear regime; the process lasts until turnaround, when the system decouples from the Hubble flow. Then, after collapse, the system forms a virialized structure. The gas inside the virialized dark matter halo then cools radiatively and collapses while conserving its angular momentum, which results in a centrifugally supported disc (White & Rees 1978; Fall &EFF-1980; White 1984). During this process the dark matter halo undergoes adiabatic contraction (Blumenthal et al. (1986)). This standard picture leads to a distribution of size and luminosity for galaxies in reasonable agreement with observations (Kauffmann 1996; Dalcanton et al. 1997; Mo et al. 1999; Avila-Reese et al. 1999; van den Bosch 2000; Dutton et al. 2007; Gnedin et al. 2007). But specific numerical simulations have revealed two problems.

The first appears in the simulations which incorporate gas, with cooling, and star formation. The gas loses a significant fraction of its angular momentum, resulting in discs which are too small in size. This problem has been termed the angular momentum catastrophe (Navarro & Benz 1991; Navarro & White 1994; Navarro & Steinmetz 1997; Sommer-Larsen et al. 1999). The cause of this problem is that, due to

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efficient cooling, the gas is accreted as dense clumps, which lose their angular momentum by dynamical friction during galaxy mergers.

The second problem is that of the angular momentum distribution. Even if angular momentum is assumed to be conserved, one cannot explain the exponential form of disc galaxies. Using cold dark matter numerical simulations (Bullock et al. (2001)), found that if discs are formed from gas with an angular momentum distribution similar to that of dark matter, this results in excess mass near the centre, in comparison with an exponential disc. Specifically, the models predict too much material with low angular momentum, and this makes it particularly difficult to explain the origin of bulgeless dwarf galaxies (van den Bosch (2001)). Responding to this difficulty Maller & Dekel (2002) proposed a model in which the angular momentum is built up by a sequence of mergers. While the acquisition of angular momentum in dissipationless N-body simulations has been well studied (Bullock et al. 2001; Vitvitska et al. 2002; Maller et al. 2002; D’Onghia & Navarro 2007; Avila-Reese et al. 2005; Bett et al. 2007, 2010), our understanding of the extent to which gas behaves similarly has only recently started to be explored via computations (Chen et al. 2003; Sharma & Steinmetz 2005; Brook et al. 2010; Kimm et al. 2011; Pichon et al. 2011; Sharma & Nath 2012). In the last decade galaxy redshift surveys (see e.g. Colless et al. (2001)) have enabled us to map the universe on increasingly large scales, and to study systematically the properties of large-scale structures. These surveys show that the galaxies are not distributed evenly, but develop within a pattern of filaments and sheets which are separated by immense voids. The later are defined observationally as large regions with very low galaxy number density, which occupy some 40% of the overall cosmic volume (Hoyle & Vogeley (2004)). It is generally believed that the voids originate from the local minima of the primordial density fluctuations, and as their mean density is lower than that of their surroundings they expand more quickly. If tidal shear plays a significant role in their development, it should not only cause them to depart from sphericity, but should entail their acquisition of spin angular momentum (Lee & Park (2006)). The specific angular momentum of cold dark matter haloes in a ΛCDM universe depends strongly on their merging histories (D’Onghia and Burkert 2004). Halos with a quiet merging history, dominated until the present epoch by minor mergers and accretion, acquire by tidal torques an average of only 3% of the angular momentum required for their rotational support. This is in direct conflict with the observational data for a sample of late-type bulgeless galaxies, which indicate that these galaxies reside in dark halos with exceptionally high values of specific angular momentum. Although minor mergers and accretion preserve or slowly increase the specific angular momentum of dark halos as a function of time, this mechanism is not efficient enough to explain the observed spin values, especially for late-type dwarf galaxies. Energetic feedback processes have been invoked to solve the problem implied by the loss of a large fraction of its specific angular momentum by the gas during infall. This is predicated on the assumption that dark halos which host bulgeless galaxies acquire their mass via quiescent accretion. D’Onghia and Burkert (2004) find an even more serious version of this problem: the specific angular momentum gained during the formation of these objects is not enough to provide rotational support even were no angular momentum lost during gas infall.

The simple tidal-torque theory predicts alignment effects of disc galaxies, as the acquisition of the angular momentum is governed partly by environmental effects, such as tidal shearing produced by the primordial matter distribution in the neighbourhood, and the moment of inertia of the protogalaxy which is forming (Schweizer (2009)). In this scenario testing the alignment of the orientations of galaxies with respect to that of their surrounding environment (filaments, sheets, and voids) provides important information about how galaxies form in the cosmic web. It is known that the angular momenta of neighbouring galaxies are correlated (Slosar et al. 2009; Lee 2011; Andrae & Jahnke 2011), indicating that the environment does influence the acquisition of angular momentum. However, Andrae & Jahnke (2011) suggested that this correlation, while plausible, is not statistically significant. In recent years, correlations between dark matter haloes and their host filaments have been predicted from N-body numerical simulations (Hatton & Ninin 2001; Faltenbacher et al. 2002; Bailin & Steinmetz 2005; Altay et al. 2006; Brunino et al. 2007; Wang et al. 2011). In particular (Aragon-Calvo et al. 2007; Trowland et al. 2012; Codis et al. 2012) found that the orientation of the halo spin vector should be mass-dependent. The spin axes of low-mass haloes tend to be aligned parallel to the filaments, whereas high mass haloes tend to adopt an orthogonal alignment. Tempel et al. (2013) used the largest current spectroscopic

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Specific stellar angular momentum $j$ against stellar mass of a sample of spiral galaxies on a logarithmic scale. Data, including confidence level of 0.04 dex, are taken from Romanowsky & Fall (2012). Full line is the best fit using our theoretical model presented in this paper (slope 1/2 and goodness of fit 0.84). We term goodness of fit the simple square root of the mean of the square deviations of each data value with respect to the theoretical linear fit. Dotted line is the best fit using the standard theory of tidal torques (slope 2/3 and goodness of fit 0.72).}
\end{figure}
galaxy redshift survey (SDSS) to study the connection between the spin axes of galaxies and the orientation of their host filaments, finding evidence that the spin axes of bright spiral galaxies have a tendency, albeit weak, to be aligned parallel to filaments while the elliptical/SO galaxies have spin axes perpendicular to the host filaments. The aim of the present article is to throw light on the origin of the angular momentum of galaxies. It is organized as follows: in Section 2 we present the observational data used as constraints on the theoretical model; in Section 3 we develop the theoretical model; in Section 4 we compare the predictions of the model with data; and in Section 5 we present our conclusions.

2 THE ANGULAR MOMENTUM OF GALAXIES: OBSERVATIONAL DATA

We have used, as the principal source of observations needed to test our theoretical model, the updated compilation of Romanowsky & Fall (2012). Motivated by a new applicable set of kinematical tracers in the outer regions of early-type galaxies (ellipticals and lenticulars), they re-examine the role of angular momentum in a range of types. They present new methods for quantifying the specific angular momentum $j$, focusing on the challenges presented by early-type galaxies, deriving firm empirical relations between stellar $j$, and mass $M$; their work is essentially an extension of the earlier studies by (Fall 1979, 1983, 2002). They carry out detailed analyses of eight galaxies with kinematical data extending as far out as 10 effective radii, and find that the observations at two effective radii are normally sufficient to estimate $j$. 

Figure 2. The data are those of Fig. 1. The dotted and full lines are the predictions of our present model (slope 1/2) fitting the extreme observed galaxies. One can see the general trend of ranges of possibilities (see also Fig. 3).

Figure 3. The data are those of Fig. 1. The dotted and full lines are the predictions of the standard model (slope 2/3) fitting the extreme observed galaxies. One can see the general trend of ranges of possibilities, and how the empty area between the two extremes is considerably greater than that of shown in Fig. 2 corresponding to our present model.

Figure 4. Specific stellar angular momentum $j$ against stellar mass of a sample of spiral galaxies (full pentagons) compared with the same parameters for early-type galaxies (empty triangles) on logarithmic scale. The data, including confidence level of 0.04 dex, are taken from Romanowsky & Fall (2012).
with high reliability. Their results show the untenability of suggestions that ellipticals could harbour large reservoirs of hidden \(j\) in their outer regions, due to angular momentum transport during major mergers. They go on to carry out a comprehensive analysis of extended kinematic data from the literature for a sample of 100 nearby bright galaxies of all morphological types, and produce a diagram of \(j\) versus \(M\). The ellipticals and spirals form two parallel tracks in \(j - M\) space, with logarithmic slopes of 0.6, which for spirals is a result related to the Tully-Fisher relation, but for the ellipticals is produced by a "conspiracy" between masses, sizes, and rotation velocities. The ellipticals contain, unsurprisingly, less angular momentum on average than spirals of equal mass, with the quantitative difference depending on the adopted value of the K-band stellar mass-to-light ratios of the galaxies. It is a factor of 3-4 if, for simplicity, mass-to-light ratio variations are neglected, but it if these are included. They decompose the spirals into discs and bulges, and find that these subcomponents follow \(j - M\) trends similar to those for the complete spirals or ellipticals.

The lenticulars show an intermediate trend, and they suggest that the morphological types of galaxies are made up of disc and bulge subcomponents, which follow separate, fundamental, \(j - M\) scaling relations. This provides a physical motivation for characterizing galaxies using two basic parameters: mass and bulge-to-disc ratio. As a next step, using an approach which is complementary to that of numerical simulations, they construct idealized models of angular momentum content in a cosmological context, using estimates of the spin and the mass of dark matter halos from a combination of empirical and theoretical model studies. They find that the width of the halo spin distribution cannot account for the differences between the ranges of \(j\) for spirals and ellipticals, but that the observations can be well reproduced if these galaxies simply retained different original fractions of their initial \(j\) complement (60% and 10%, respectively). They consider a number of physical mechanisms which could account for the simultaneous evolution of \(j\) and \(M\), including outflows, stripping, collapse bias, and merging, emphasizing that the vector sum of all such processes must give rise to the observed \(j - M\) relations. They suggest that a combination of early collapse and multiple mergers (major or minor) may account naturally for the trend observed for the ellipticals. More generally, the observed variations in angular momentum provide straightforward but fundamental constraints for any scenario which aims to model galaxy formation. We use in the present study the diagram introduced by Fall (1983): a general diagram of \(j\) versus stellar mass, \(M\), where \(j\) is defined by \(j = J/M\), i.e. it is the stellar specific angular momentum parameter. This diagram has the important properties that it deals with conservative physical quantities, and that the axes represent independent variables. The \(M\) axis gives a mass scale, while the \(j\) axis is the equivalent of a length scale times a rotation velocity scale. On the contrary the often used relation between \(M\) and the circular velocity \(v_c\) involves correlated variables, since \(v_c\) may be directly related to \(M\) (Tully & Fisher 1977; Dutton et al. 2010; Trujillo-Gomez et al. 2011). On the other hand, Trujillo et al. (2006) have found from two large galaxy redshift surveys (2dFGRS and SDSS) that at the 99.7% confident level spiral galaxies located on the shells of the largest cosmic voids have rotation axes that lie preferentially on the

![Figure 5. Specific stellar angular momentum \(j\) against stellar mass of a sample of early-type galaxies on a logarithmic scale. Data, including confidence level of 0.04 dex, are taken from Romanowsky & Fall (2012). Full line is the best fit using the theoretical model presented in this paper (slope 1/2 and goodness of fit 0.53). Dotted line is the best fit using the standard theory of tidal torques (slope 2/3 and goodness of fit 0.61). One can see how neither model gives a good fit for the early-type galaxies of this sample, in fact the best fit suggests a slope close to 0.9.](image)
togalaxy, we find that the trajectory of a test particle inside a given protogalaxy is
\[ \mathbf{r}(q, t) = a(t)x(q, t) = a(t)[q - \beta(t)[-2\Omega \times \mathbf{v}(q, t)]d\mathbf{q} \] (1)
where \( q \) is a Lagrangian coordinate defined as the \( x \) position of the particle at \( t \rightarrow 0 \), \( \Omega \) is the mean angular velocity of the voids, \( M \) is the mass of the protogalaxy, \( d \ll q \) is the distance travelled by the gas during collapse, and \( i \) is the unit vector in the radial direction of the protogalaxy. The specific angular momentum of the material composing the protogalaxy may be written as
\[ \mathbf{j}(t) = \frac{J(t)}{M} = \frac{1}{M} \int_{V_L} [\mathbf{r}(q, t) - \mathbf{r}_1(t)] \times \mathbf{v}(q, t) d^3q \] (2)
and as a first approximation we have
\[ \mathbf{j}(t) = \frac{-\rho_0 a^4 b}{M} \int_{V_L} (\mathbf{q}_M) \times (-2\Omega \times \mathbf{v}(q, t)) d^3q \] (3)
where \( b' \) is the first time derivative of \( b \), \( \mathbf{r}_1 \) is the position of the centre of mass, and \( \mathbf{q}_M \) is \( q \) measured with respect to the centre of mass of the protogalaxy. We then derive:
\[ \mathbf{j}(t) = \frac{2n_0 a^4 b'}{M} \int_{V_L} \mathbf{q}_M (\mathbf{h} \cdot \mathbf{v}) d^3q \] (4)
where \( n_0 \) is the unit vector in the direction of the centre of rotating void. We then find:
\[ \mathbf{j}(t) \propto nM^{1/2} \] (5)
and this last result is independent of symmetry adopted, i.e. the same expression holds whether we take spherical, cylindrical, or linear coordinates. If we use the same approximation in Tidal Torque Theory (TTT), we find:
\[ \mathbf{j}(t) \propto \frac{1}{M} \int_{V_L} \mathbf{q}_M \mathbf{h} \frac{M_G}{r_0} d^3q \] (6)
where \( \mathbf{h} \) is the unit vector in the direction perpendicular to the gradient of the tidal potential, \( M_G \) is the mass of the nearest protogalaxy, and \( r_0 \) is the distance to the centre of mass of the nearest protogalaxy. We then have, for simple TTT:
\[ \mathbf{j}(t) \propto \mathbf{h} M^3 \] (7)
where \( \beta \) takes different values for different symmetries: 1 for linear coordinates, 1/2 for cylindrical coordinates, and 1/3 for spherical coordinates. However in simple TTT this relation does not in itself constitute a problem. The principal difficulties with the usual version of TTT are produced, as pointed out in the Introduction, by the loss of angular momentum by the predicted discs, which in consequence are too small, and the failure to offer a natural explanation of the exponential distribution of mass within a disc. These problems could be surmounted in our scenario because the specific angular momentum of a given galaxy is not conserved. There is a supply of angular momentum from the Coriolis force in the void. This yields the result that although \( \Omega \) decreases due to dynamical friction on a cosmological timescale \( (10^{10} \text{ yr}) \), the mass of the protogalaxy and \( \mathbf{d} \) increase systematically on a galactic timescale \( (10^9 \text{ yr}) \). We assume that cosmic voids acquire their angular momentum by tidal torque gravitational interaction, and so we calculate the angular momentum of a typical cosmic void at the time of maximum expansion of a spherical overdensity following Catelan & Theuns (1996)
\[ R^2M_T \Omega \approx 2.4 \times 10^{66} \left( \frac{h}{0.5} \right)^{-2} \left( \frac{1 + z}{4} \right)^{-1/2} A(M_T) \] (8)
where \( A(M_T) = \left( \frac{M_T}{10^{12} M_\odot} \right)^{1/3} \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1} \) and we have changed the total mass of a protogalaxy (including dark matter and divided by 10) for the mass of the cosmic void \( M_T \) (including dark matter and divided by 10). \( R \) is the radius of the void, and \( \Omega \) is the angular velocity of void. The evolution of the angular momentum \( L \) of a galaxy in the surface of the void will be, following Pfeffer (1965) and assuming no pressure, no friction and no tangential velocity inside the protogalaxy:
\[ \frac{\partial L}{\partial t} = \int_V \rho f R_G dV \] (9)
where \( V \) is the volume of the galaxy, \( R_G \) the radius of the galaxy, \( \rho \) the density of the galaxy, \( v \) the radial velocity of matter inside the galaxy, and \( f = 2\Omega \sin \theta \), where \( \theta \) is the angular coordinate of the galaxy in the surface of the void (0 at the equator and \( \pi \) at the rotation axis).
From equation (8) we have \( \Omega \propto M_T^{1/3} \), and from equation (9) we have

\[ \begin{aligned}
\text{Figure 6. Specific stellar angular momentum } j \text{ against stellar mass } M \text{ on logarithmic scale. Data are those of Fig. 4. Dotted line is our model best fit for spirals (slope 1/2 and goodness of fit 0.84). Full line is the prediction of our model for ellipticals assumed as results of interactions between spirals (slope 0.85 and goodness of fit 0.78), because the lower } j \text{ implies that more elliptical should form near the equatorial plane of the original voids where the original spirals formed by Coriolis effect, and so the probability of interactions with other spirals pertaining to other neighbouring voids is greater than that for spirals near the rotation axis. We have assumed that this probability is proportional to the relative length rotated, i.e. proportional to the angle } 2\pi \text{ for the equator and proportional to 1 for the rotation axis.}
\end{aligned} \]
The same as in Fig. 7 but here the two extreme theoretical model slopes 2/3 following the standard theory. We can see that now the fit is again not so good as that of our present model shown in Fig. 7.

Figure 7. The same as in Fig. 2, but here for the early-type galaxies and the two extreme theoretical model slopes 0.85. The two galaxies showing log $j$ below 1 are not taken into account following the original authors Romanowsky & Fall (2012), although one of them is in fact very near our fit.

Figure 8. The same as in Fig. 7 but here the two extreme theoretical model slopes 2/3 following the standard theory. We can see that now the fit is again not so good as that of our present model shown in Fig. 7.

\[
\frac{\partial L}{\partial t} \simeq MG2\Omega \sin\theta R_G v
\]  

(10)

Then, from equations (8) and (9) we have

\[
\frac{\partial L}{\partial t} \propto MG M_T^{2/3} \sin\theta R_G v
\]  

(11)

and assuming free fall of matter to the centre of a galaxy, i.e. $v = \sqrt{\frac{2GM}{R_G}}$, where $d \ll R_G$ is the displacement of matter in free fall, we have

\[
\frac{\partial L}{\partial t} \propto MG M_T^{2/3} \sin\theta M_G^{1/2}
\]  

(12)

and due to $j = \frac{L}{MG}$, and assuming $j_0 = 0$ we have

\[
j \propto M_T^{2/3} \sin\theta M_G^{1/2}
\]  

(13)

So, we have a range of values of $j$ from zero for galaxies formed at the equator of the void to $j_{\text{max}}$ for galaxies formed at the rotation axis of the void. In order to compare with observational data we will calculate the value of $L$ predicted by our model for a typical galaxy similar to the Milky Way. From equation (10) and assuming $L(t = 0) = 0$, we obtain

\[
L \simeq MG2\Omega \sin\theta R_G v t
\]  

(14)

where

\[
t = \frac{2}{3 \times 1.6 \times 10^{-18}(1 + z)^{3/2}}
\]  

(15)

(taking $H_0 = 500\, \text{km}\, \text{s}^{-1}\, \text{Mpc}^{-1}$ and $\Omega_0 = 1$) means the cosmic time in a standard cosmology and $z$ is the redshift.

So, from equations (8) and (14) we have

\[
L \simeq \frac{M_G2 \times 2.4 \times 10^{66}}{R_T^2 M_T} \left(\frac{1 + z}{4}\right)^{-1/2} A(M_T) \sin\theta \sqrt{2GM_G d t}
\]  

(16)

Then using for $t$ equation (15) and taking $h = 0.5$, $d = 1\, \text{kpc}$, $R = 10\, \text{Mpc}$, $M_T = 10^{15} M_\odot$ and $z = 0$ we obtain a value for the angular momentum of the sample Milky Way $L = 1 \times 10^{66} \sin\theta \, \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$, which is very similar (for the case $\theta = \frac{\pi}{2}$) to that obtained observationally for the real Milky Way $L = 1.8 \times 10^{67} \, \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$. In our scenario, the early-type galaxies originate mainly via gravitational interaction between spirals. Their lower values of $j$ imply that these interactions were favoured closer to the equatorial planes of the original voids, where the original spirals formed as a result of the Coriolis effect have a probability of interacting with spirals pertaining to a neighboring void which is higher than those spirals which formed nearer to the void rotation axis. If we take the probability of interaction to be proportional to the relative orbital length rotated, i.e. to $2\pi$ for equatorially formed galaxies and to 1 for galaxies near the pole, our model gives a slope of 0.85 for the relation log $j$/log $M$. The value precisely unity for the void rotation pole is because the solid angle over which a void sees the neighboring void at their closest approach is $\frac{\pi r^2}{4}$ which is the radius of each void.

From eq. (13) we have $j$ as a function of a factor $\sin\theta$ with $0 \leq \theta \leq \pi/2$. So, in our picture we have galaxies with $j$ between almost zero and a certain value $j_{\text{max}}$ close to that observed for the Milky Way) depending on the place of formation. Zero for galaxies formed at the equator of the void and $j_{\text{max}}$ for those galaxies formed near the rotation axis of the void. So, the problem of accounting for the observed angular momentum distribution of discs of dwarf galaxies (van...
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den Bosch (2001)) is naturally explained in our context by considering dwarf galaxies as the result of mergers of galaxies formed at different places (different $\theta$ and then different $j$). Then, in a given galaxy, the probability of observing $j$ near zero will be very similar to the probability of observing $j$ near $j_{\text{max}}$, as is observed (van den Bosch (2001)). On the other hand, the same factor $sin\theta$ could explain the wide spread among $log(j)$ for spirals with the highest $j$ and ellipticals with the lowest $j$ with similar masses (see Fig. 6). The reason is that spirals have formed preferentially near the rotation axis of voids (where $\theta = \pi/2$) and ellipticals near the equator (where $\theta = 0$). Regarding the orientation of galactic spins we assume that both filaments and sheets are contact surfaces between two voids, and so we find that the galaxies pertaining to those structures suffer gravitational influences from both voids each one in the direction of the centre of each void. Although the surface on which the few void should have original spin directions perpendicular to the void surface due to the Coriolis effect, only those galaxies which are near the line connecting the centres of two voids still show this primaeval spin direction because the gravitational effects of the two voids align precisely along this direction (where, in fact, there is the highest probability of gravitational hard interaction between galaxies because it is the zone of closest proximity between voids, so that this is where it is most probable that there is transformation from spiral to elliptical/SO galaxies. This is in agreement with the observed correlation between ellipticals and their spin direction perpendicular to filaments and sheets). However, those galaxies (the majority) that are far from the line connecting the centres of voids are subject to gravitational influences of two centres of voids subtending angles which are more and more acute as one goes away from the greatest proximity point between voids, and so the vectorial net (resultant) gravitational influence points increasingly in the observed direction, i.e. parallel to the filaments and sheets (see Fig. 9). Even if one assumes that the power spectrum of $\lambda$ spin parameter (Peebles (1969)) is proportional to the power spectrum of $j$, which is not very realistic because $\lambda = \frac{E j^{1/2}}{M \lambda}$ with $E$ the total energy and $M$ the mass, our model predicts a functional relation $j \propto sin\theta$ from equation (13) at the time of galaxy formation, due to the Coriolis force induced by the rotation of cosmic voids. But to test our model with observations one must take into account that the probability of $j$ observed is proportional to $sin\theta cos\theta$ after the rotation of a void interacting with another void to form the observed sheets and filaments, because the galaxies formed near the equator of the void have $2\pi R_c$ of length in which to interact directly with galaxies with the other void, while galaxies formed at other latitudes have values $2\pi R_c cos\theta$. So the maximum is skewed to lower $j$ as the void rotate, which is observed using large SDSS samples of tens of thousands of galaxies and also obtained in numerical simulations (Hernandez et al. 2007; Berta et al. 2008; Cervantes-Sodi et al. 2008). Other causes such as non-sphericity, multiple voids connecting with one void, etc., could displace the peak of $P(j)$ to lower $j$ as is observed as a log-normal distribution. However, one must be very cautious with the log-normal distribution function, because that skewed distribution describes data from many diverse disciplines, because the fluctuations which arise during the course of evolution toward more probable states yield multiplicative variations about the mean which is the main characteristic of this type of probability distribution; one example is the logarithmic function of the symmetric distribution which represents additive variables through the Central Limit Theorem. In considering timescales involved we have, from eq. (8) and $\Omega = \frac{\omega}{c}$ where $\omega$ is the characteristic period of a cosmic void rotation, that $T_V \approx 10^{11}$ yr, which is several orders of magnitude greater than the collapse time of a galactic structure. This implies that our mechanism is, to first order, of great interest for explaining the origin of angular momentum of galaxies.

4 COMPARISON OF THE MODEL PREDICTIONS WITH OBSERVATIONS

In Fig. 1 we show the specific stellar angular momenta, $j$ of a sample of spiral galaxies plotted against stellar mass, both on logarithmic scales. The data are taken from Romanowsky & Fall (2012). The full line is the best fit obtained using the theoretical model presented in section 2, with a slope of 1/2 and with a goodness of fit of 0.84. The dotted line is the best fit using the standard theory of tidal torques, with a slope of 2/3 and a goodness of fit of 0.72. In Fig. 2, using the same data set we show model fits (slope 1/2) to the extreme values for the observed galaxies, and in Fig. 3 we show the fits for the standard model (slope 2/3) to the extreme values of the observed galaxies. Figs. 1, 2 and 3, taken together, show that our model predictions give significantly improved fits to the data. In Fig. 4 we have plotted log $j$ against log $M$ for the spiral galaxies (filled pentagons) and for early-type galaxies (open triangles) (Romanowsky & Fall (2012)), and in Fig. 5 we have plotted the early-type galaxies alone (filled triangles) together with linear fits using our model (full line and goodness of fit 0.53), and the standard tidal torque model (dotted line and goodness of fit 0.61). It is clear that neither model gives an adequate fit to the early type galaxies in the sample; the best fit would have a slope of close to 0.9. In Fig. 6 the best fit for the spirals in the sample, using our model, is the dotted line of slope 1/2 and goodness of fit 0.84, while the full line shows the result of our model for ellipticals, when we have taken into account their production due to the interactions between spirals, which yields a slope of 0.85 and a goodness of fit 0.78. It is clear that the confidence levels of the observational data (0.04 dex) does not constitute a significant restriction to the fits. The increase of the slope which occurs in our model is because a lower $j$ implies that the galaxy was nearer to the equatorial plane of the original voids, where the original spirals were affected by the Coriolis force of greatest amplitude. The probability of interaction with other spirals belonging to a neighboring void is progressively greater the nearer the spiral originally was to the equator of the void. In Fig. 7 we have plotted the early-type galaxies alone with the two plots for the extreme cases plotted with the slope predicted by our model: 0.85. The two galaxies having values of log $j$ less than 1 are, following Romanowsky & Fall (2012), considered not to be significant. In Fig. 8 we show the same galaxies with predictions for the standard model, having slopes of 2/3. Here, as for the spirals, our model gives a better account of the measurements.
though galaxies on the surface of each void should have orig-
cancellation of the angular momentum, as is observed. Al-
lieves similarly situated in the neighboring void, transforms a
have the greatest probability of interaction with other galax-
the galaxies farthest from the rotation axis are those which
tidal interaction between quasi-spherical voids, in which
the angular momentum of the spiral galaxies. Following this,
Coriolis force due to the rotation of the voids gave rise to
gravitational collapse of the gas into protogalaxies and the
torque interactions, and subsequently the combination of the
around bubbles or voids, these acquired rotation by tidal
gravitational influences supply a net force which is always in
the direction of the interaction surface.

5 CONCLUSIONS

This study presents prima facie evidence that the origin of
the angular momentum of galaxies involves processes which
have not previously been taken into account. The most re-
cent observational data could indicate a scenario in which,
after recombination, when the matter became organized
around bubbles or voids, these acquired rotation by tidal
torque interactions, and subsequently the combination of the
gravitational collapse of the gas into protogalaxies and the
Coriolis force due to the rotation of the voids gave rise to
the angular momentum of the spiral galaxies. Following this,
the tidal interaction between quasi-spherical voids, in which
the galaxies farthest from the rotation axis are those which
have the greatest probability of interaction with other galax-
ies similarly situated in the neighboring void, transforms a
fraction of the spirals into ellipticals, with the consequent
cancellation of the angular momentum, as is observed. Al-
though galaxies on the surface of each void should have orig-
inal spin directions perpendicular to the void surface due to
the Coriolis effect, only those galaxies which are near the line
connecting the centres of two voids still show this primae-
val spin direction because the opposing gravitational effects
of the two voids point precisely in this direction (where,
in fact, there is the highest probability of strong grava-
tional interaction between galaxies because it is the zone of
closest proximity of the two voids, and therefore where the
transformation from spiral to elliptical/SO galaxies is most
probable. This is in agreement with the observed correlation
between ellipticals and their spin direction perpendicular to
filaments and sheets. However, those galaxies (spirals) that
are far from the line connecting the centres of voids have
gravitational influences of two centres of voids subtending
angles which are increasingly acute as one goes away from
the zone of proximity between voids, and so the vectorial
net (resultant) gravitational influence is aligned increasingly
in the observed direction, i.e. parallel to the filaments and
sheets. However, we note here that this point is nowadays
under observational controversy, and there are authors who
find the spin axis for spirals showing a weak tendency to be
perpendicular to filaments (Varela et al. 2012; Zhang et al.
2014). We have shown here that using this scenario we can
give an account similar to that of the standard tidal torque
model of the observed dependence of the specific angular
momentum on the mass of both spirals and ellipticals, and
also of the observed spin orientations. Although Cervantes-
Sodi et al. (2010) found a weak correlation between the spin
magnitude of neighbouring galaxies, but not between their
orientations, suggesting that the original cause of angular
momentum for each pair of galaxies could have been similar,
but the redistribution of angular momentum at later stages
of evolution was important. Also, similar inertial effects due
to the Coriolis force will appear if the collapse time of a galac-
tic structure $T_C$ is greater than $\frac{\Omega}{2\pi}$, i.e. if $T_C$ is long in
comparison to the rate of change of the void’s spin. In ad-
dition due to the centrifugal force, if rotation increases with
distance from the centre of the void due to tidal torques,
because this increasing over the extent of the forming galaxy
it will result in the collapsing proto-galaxy acquiring some
angular momentum parallel to the overall void spin. But
to study these two possibilities we would need more pre-
cise data than those we have now, because both of them
imply second order approximation instead of the first order
approximation implied by our model. The model also of-
fers a natural explanation for the distribution of the specific
angular momentum in dwarf galaxies, where the probabil-
ity of finding angular momentum close to zero is compar-
able with the probability of finding the angular momentum
close to its maximum allowed value (van den Bosch (2001)).
This would arise if the galaxies were formed by mergers of
smaller units distributed from the equator to the pole of the
rotating void. The scenario proposed here would also give
rise to ellipticals and spirals having similar masses but with
the latter having much greater angular momentum than the
former, since the spirals would originate preferentially from
near the void rotation axis, while the ellipticals would tend
to originate nearer the equator. The further steps needed to
test this new scenario include observations of the alignment
of galactic spins, and also numerical simulations incorpo-
rating the effect of the Coriolis force as indicated conceptu-
ally here. The present study shows an additional mechanism
contributing to the development of galaxy angular moment-
num which supplements the well established Tidal Torque
Theory, and should be taken into account when confronting
observations.

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