Cut-off wavenumber of Alfvén waves in partially ionized plasmas of the Solar Atmosphere

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Outline

Introduction

- The presence of a cut-off wavenumber in fully ionized resistive single-fluid MHD has been reported in several classical textbooks
- For instance, Chandrasekhar (1961) describes the behaviour of Alfvén waves in a viscous and resistive medium. In this case, the Alfvén wave frequency, ω, is given by:

$$\omega = \pm k\sqrt{\left[V_A^2 - \frac{1}{4}(\nu-\eta)^2 k^2\right]} + \frac{1}{2} i(\nu+\eta)k^2 \qquad (1)$$

- where V_A is the Alfvén speed, ν the kinematic viscosity, η the magnetic diffusivity, and k the wavenumber
- This expression clearly points out that for a value of the wavenumber such as,

$$k = \pm\frac{2V_A}{\nu-\eta} \qquad (2)$$

- the real part of the frequency becomes zero, and only the imaginary part of the frequency remains

Introduction

- The cut-off wavenumber means that waves with a wavenumber higher than the cut-off value are evanescent
- On the other hand, the real part of the Alfvén frequency in Eq.(1) can be written as:

$$\omega_r = \pm k V_A \sqrt{1 - \frac{1}{4V_A^2}(\nu - \eta)^2 k^2} = \pm k \Gamma_A \tag{3}$$

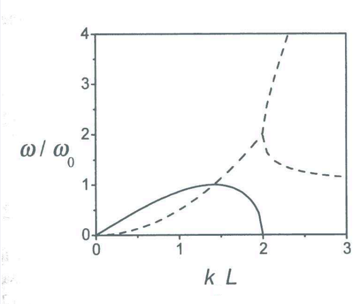
- with,

$$\Gamma_A = V_A \sqrt{1 - \frac{1}{4V_A^2}(\nu - \eta)^2 k^2} \tag{4}$$

- representing a modified Alfvén speed which goes to zero for the cut-off wavenumber, i. e. the wave ceases its propagation
- Chandrasekhar (1961) did not make any explicit comment about the presence of this cut-off wavenumber

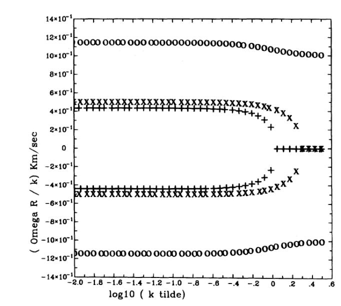
Introduction

- Ferraro & Plumpton (1961) and Kendall & Plumpton (1964) considered the effects of finite conductivity on hydromagnetic waves, and showed that for $\eta k < 2V_A$, we have time damped waves, while for $\eta k > 2V_A$ there is no wave propagation at all
- Furthermore, Cramer (2001) also pointed out the same effect, showing that when the wavenumber becomes greater than $2R_m/L$, where R_m is the magnetic Reynolds number and L a reference length, the real part of the Alfvén wave frequency becomes zero (Solid line Figure below)



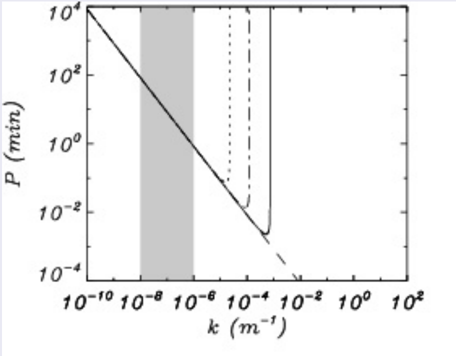
Introduction

- Significant parts of the solar atmosphere, namely photosphere, chromosphere and prominences, as well as other astrophysical environments, are made of partially ionized plasmas
- In the astrophysical context, Balsara (1996) studied MHD wave propagation in molecular clouds using the single-fluid approximation, and cut-off wavenumbers appeared for Alfvén and fast waves



Introduction

- Forteza et al. (2008), Barceló et al. (2011), Soler et al. (2009a, 2009b) and Soler et al. (2011) used the single-fluid approximation to study the damping of MHD waves produced by ion-neutral collisions in unbounded and bounded medium with prominence physical properties



Introduction

- They found that the cut-off wavenumber for Alfvén waves is given by,

$$k = \pm \frac{2 V_A \cos\theta}{(\eta_c \cos^2\theta + \eta \sin^2\theta)} \tag{5}$$

- with θ the propagation angle with respect to the magnetic field, and η_c the Cowling's diffusivity. In this case, the modified Alfvén speed (Barceló et al. 2011) is given by,

$$\Gamma_A = V_A \sqrt{1 - \frac{(\eta_c \cos^2\theta + \eta \sin^2\theta))^2 k^2}{4 V_A^2 \cos^2\theta}} \tag{6}$$

- For fully ionized resistive plasmas, $\eta = \eta_c$ and, for parallel propagation, we recover the Ferraro & Plumpton (1961) and Kendall & Plumpton (1964) cut-off wavenumber
- Finally, Singh & Krishnan (2010) studied the behaviour of Alfvén waves in the partially ionized solar atmosphere and they also reported about the cut-off wavenumber

Introduction

- Summarizing, a cut-off wavenumber appears when the single-fluid approximation is used to study MHD waves in partially ionized or resistive astrophysical plasmas
- However, up to now, an explanation for the cut-off wavenumber is missing, and this topic is relevant in connection with MHD waves in solar partially ionized plasmas such as spicules, prominences, chromosphere and photosphere
- Then, what causes the appearance of the cut-off wavenumber in the single-fluid approximation?

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Alfvén waves: Main Equations

- We study partially ionized plasmas made of electrons (e), ions (i) and neutral (hydrogen) atoms (n)
- Linearized fluid equations for each species can be split into parallel and perpendicular components of the perturbations with respect to the unperturbed magnetic field
- Since we are interested in Alfvén waves, incompressible plasma and perpendicular components are considered
- For time scales longer than ion-electron and ion-ion collision times, the electron and ion gases can be considered as a single fluid
- Then, in the two-fluid description one component is the charged fluid (electron+protons) and the other component is the gas of neutral hydrogen (Zaqarashvili et al. 2011)

Alfvén waves: Main Equations

- Next, we may go a step further and derive the single-fluid MHD equations. We use the total velocity (i.e. velocity of center of mass)

$$\vec{u}_\perp = \frac{\rho_i \vec{u}_{i\perp} + \rho_n \vec{u}_{n\perp}}{\rho_i + \rho_n} \tag{7}$$

- the relative velocity

$$\vec{w}_\perp = \vec{u}_{i\perp} - \vec{u}_{n\perp}. \tag{8}$$

- and the total density

$$\rho = \rho_i + \rho_n, \tag{9}$$

Alfvén waves: Main Equations

- Then, our Equations are:

$$\rho\frac{\partial\vec{u}_\perp}{\partial t}=\frac{1}{4\pi}(\nabla\times\vec{b}_\perp)\times\vec{B}, \tag{10}$$

$$\frac{\partial\vec{w}_\perp}{\partial t}=\frac{1}{4\pi\rho\xi_i}(\nabla\times\vec{b}_\perp)\times\vec{B}+\frac{c\alpha_{en}}{4\pi e n_e\rho\xi_i\xi_n}\nabla\times\vec{b}_\perp-\frac{\alpha_{in}+\alpha_{en}}{\rho\xi_i\xi_n}\vec{w}_\perp, \tag{11}$$

$$\frac{\partial\vec{b}_\perp}{\partial t}=\nabla\times(\vec{u}_\perp\times\vec{B})-\eta\nabla\times(\nabla\times\vec{b}_\perp)-\frac{c}{4\pi e n_e}\nabla\times\left((\nabla\times\vec{b}_\perp)\times\vec{B}\right)+$$
$$+\frac{c\alpha_{en}}{e n_e}\nabla\times\vec{w}_\perp+\xi_n\nabla\times\left(\vec{w}_\perp\times\vec{B}\right), \tag{12}$$

Alfvén waves: Main Equations

- where $\xi_i = \rho_i/\rho$, $\xi_n = \rho_n/\rho$
- and

$$\eta = \frac{c^2}{4\pi\sigma} = \frac{c^2}{4\pi e^2 n_e^2}\left[\alpha_{ei} + \frac{\alpha_{ei}\alpha_{en}}{\alpha_{ei} + \alpha_{en}}\right] \tag{13}$$

- The single-fluid Hall MHD equations are obtained from Eqs. (10-12) as follows:
- The inertial term (the left hand-side term in Eq.11) is neglected and $\vec{w}_\perp$, defined from Eq. (11), is substituted into the Induction equation
- Then, we obtain the Hall MHD equations

Alfvén waves: Main Equations

- $$\rho \frac{\partial \vec{u}_\perp}{\partial t} = \frac{1}{4\pi}(\nabla \times \vec{b}_\perp) \times \vec{B}, \tag{14}$$

$$\frac{\partial \vec{b}_\perp}{\partial t} = \nabla \times (\vec{u}_\perp \times \vec{B}) - \frac{c}{4\pi e n_e}\left[1 - \frac{2\xi_n \alpha_{en}}{\alpha_{in} + \alpha_{en}}\right] \nabla \times \left((\nabla \times \vec{b}_\perp) \times \vec{B}\right) + \\ + \eta_c \nabla^2 \vec{b}_\perp, \tag{15}$$

- where

$$\eta_c = \eta + \frac{\xi_n^2 B^2}{4\pi \alpha_{in}}, \tag{16}$$

- is the Cowling's coefficient of magnetic diffusion and the second term in the right-hand side of Eq.(15) is the Hall current term modified by electron-neutral collisions

Alfvén waves: Main Equations

- The usual single-fluid MHD equations, which are widely used for description of Alfvén waves in partially ionized plasmas, are obtained from Eqs. (14-15) after neglecting the modified Hall term in Eq. (15)

$$\rho\frac{\partial\vec{u}_\perp}{\partial t}=\frac{1}{4\pi}(\nabla\times\vec{b}_\perp)\times\vec{B}, \tag{17}$$

$$\frac{\partial\vec{b}_\perp}{\partial t}=\nabla\times(\vec{u}_\perp\times\vec{B})+\eta_c\nabla^2\vec{b}_\perp \tag{18}$$

Equilibrium background and Procedure

- Unbounded and homogeneous medium with physical properties akin to quiescent solar prominences, and the unperturbed magnetic field $\vec{B}$ is directed along the z axis of cartesian frame. Next, we consider the Alfvén wave propagation along the magnetic field, consequently we perform the Fourier analysis with $exp(-i\varpi t+ikz)$, where ϖ is the wave frequency and k is the wavenumber.

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Dispersion Relations and Results

First set of Equations

- $$\rho\frac{\partial \vec{u}_\perp}{\partial t} = \frac{1}{4\pi}(\nabla\times\vec{b}_\perp)\times\vec{B}, \tag{19}$$

$$\frac{\partial \vec{w}_\perp}{\partial t} = \frac{1}{4\pi\rho\xi_i}(\nabla\times\vec{b}_\perp)\times\vec{B} + \frac{c\alpha_{en}}{4\pi e n_e\rho\xi_i\xi_n}\nabla\times\vec{b}_\perp - \frac{\alpha_{in}+\alpha_{en}}{\rho\xi_i\xi_n}\vec{w}_\perp, \tag{20}$$

$$\frac{\partial \vec{b}_\perp}{\partial t} = \nabla\times(\vec{u}_\perp\times\vec{B}) - \eta\nabla\times(\nabla\times\vec{b}_\perp) - \frac{c}{4\pi e n_e}\nabla\times\left((\nabla\times\vec{b}_\perp)\times\vec{B}\right) + + \frac{c\alpha_{en}}{e n_e}\nabla\times\vec{w}_\perp + \xi_n\nabla\times\left(\vec{w}_\perp\times\vec{B}\right), \tag{21}$$

Dispersion Relations and Results

First Dispersion Relation

- From Eqs. (19-21) the following dispersion relation is obtained,

$$a\delta^2\nu\left[1+(1+\nu)\zeta\right]\omega - a\xi_n\left[(\pm a+\xi_i\omega)\omega-1\right]\omega + i\delta\Big[a^2\xi_n\omega^2+$$
$$+\nu\Big[\pm a\omega(\zeta-1)-a^2\zeta\omega^2+\xi_i(1+\zeta(1\mp 2a\omega)+((a^2-1)\zeta-1)\omega^2)\Big]\Big]=$$
$$=0, \tag{22}$$

- where $\omega = \varpi/(kV_A)$, $\tau = \omega_e/\omega_i$, $a = kV_A/\omega_i$, $\delta = \delta_{ei}/\omega_e$, $\nu = \alpha_{in}/\alpha_{ei}$ and $\zeta = \alpha_{en}/\alpha_{in}$. Here $V_A = B/\sqrt{4\pi\rho}$ is the Alfvén speed, $\delta_{ei} = \alpha_{ei}/(m_e n_e)$ is the electron-ion collision frequency, $\omega_i = eB/(cm_i)$ and $\omega_e = eB/(cm_e)$ are ion and electron giro-frequencies respectively.
- The dispersion relation in Eq. (22) has been solved numerically and the solution is plotted in Fig. a) which shows that there is no cut-off wavenumber

Dispersion Relations and Results

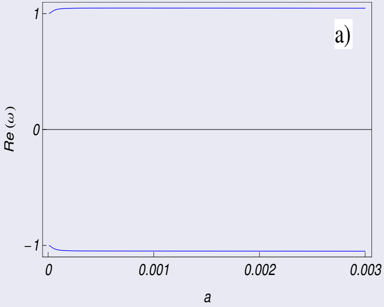

Figure: Real part of the dimensionless wave frequency $Re(\omega)$ versus the dimensionless Alfvén frequency $a = kV_A/\omega_i$ in partially ionized plasmas, where ω_i is the ion gyro-frequency. a) Single-fluid MHD equations with inertial term

Dispersion Relations and Results

Single-fluid Hall MHD Equations

- $$\rho \frac{\partial \vec{u}_\perp}{\partial t} = \frac{1}{4\pi}(\nabla \times \vec{b}_\perp) \times \vec{B}, \tag{23}$$

$$\frac{\partial \vec{b}_\perp}{\partial t} = \nabla \times (\vec{u}_\perp \times \vec{B}) - \frac{c}{4\pi e n_e}\left[1 - \frac{2\xi_n \alpha_{en}}{\alpha_{in} + \alpha_{en}}\right] \nabla \times \left((\nabla \times \vec{b}_\perp) \times \vec{B}\right) + \\ + \eta_c \nabla^2 \vec{b}_\perp, \tag{24}$$

Dispersion Relations and Results

Second Dispersion Relation

- The dispersion relation is given by,

$$\delta^2\nu^2\xi_i^2(1+\zeta)^2\omega^4-2\delta^2\nu^2\xi_i^2[1+\zeta(2+\zeta)]\omega^2-\Big[\delta^4\nu^2[1+(1+\nu)\zeta]^2+\xi_n^4+$$

$$+2\delta^2\nu(1+\zeta)\xi_n^2+\delta^2\nu^2\Big(1+\zeta^2(1-4\xi_i)+2\xi_i^2(1+2\zeta)\Big)\Big]a^2\omega^2+$$

$$+\delta^2\nu^2(1+\zeta)^2\xi_i^2+2ia(1+\zeta)\xi_i\delta\nu\left[\delta\nu(1+(1+\nu)\zeta)+\xi_n^2\right]\omega(\omega^2-1)=$$

$$=0. \tag{25}$$

- The numerical solution of this dispersion relation is shown in Fig. b), and such as it can be seen in Fig. d), the real part of the frequency never becomes zero. Thus, the single-fluid approach in partially ionized plasmas with Hall current term does not include a cut-off wavenumber.

Dispersion Relations and Results

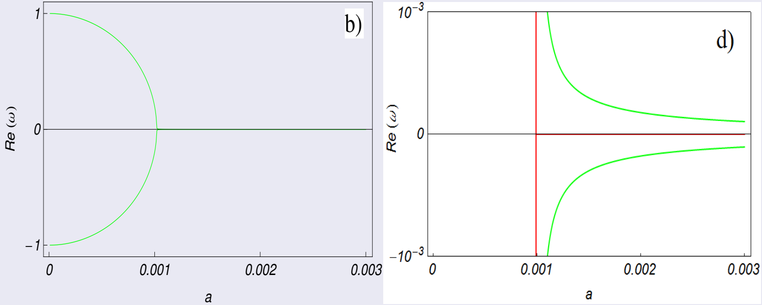

Figure: Real part of the dimensionless wave frequency $Re(\omega)$ versus the dimensionless Alfvén frequency $a = kV_A/\omega_i$ in partially ionized plasmas. a) Single-fluid Hall MHD equations; d) Zoom of the solution in Figure b) (green line) near x axis.

Dispersion Relations and Results

Third Dispersion Relation

- Dispersion relation (25) can be significantly simplified as $\zeta \ll 1$ i.e. $\alpha_{en} \ll \alpha_{in}$ and it becomes (Pandey and Wardle, 2008),

$$\omega^2 + \left[i\frac{\eta_c k}{V_A} \pm \frac{kV_A}{\omega_i \xi_i} \right] \omega - 1 = 0, \tag{26}$$

- whose analytical solution is given by,

$$\omega = \frac{1}{2} \left[-\left(\frac{ik\eta_c}{V_A} \pm \frac{kV_A}{\xi_i \omega_i} \right) \pm \sqrt{4 + \left(\frac{ik\eta_c}{V_A} \pm \frac{kV_A}{\xi_i \omega_i} \right)^2} \right] \tag{27}$$

- Equation (27) clearly shows that the wave frequency has always a real part i.e. the presence of the Hall current term (the second term in front of ω in Eq. 26) forbids the appearance of a cut-off wavenumber

Dispersion Relations and Results

Single-fluid MHD Equations

- Finally, single-fluid MHD equations are considered,

$$\rho\frac{\partial \vec{u}_\perp}{\partial t} = \frac{1}{4\pi}(\nabla \times \vec{b}_\perp) \times \vec{B}, \tag{28}$$

$$\frac{\partial \vec{b}_\perp}{\partial t} = \nabla\times(\vec{u}_\perp \times \vec{B}) + \eta_c \nabla^2 \vec{b}_\perp \tag{29}$$

Dispersion Relations and Results

Fourth Dispersion Relation

- The dispersion relation of Alfvén waves commonly used in partially ionized plasmas is:

$$\omega^2 + i\frac{\eta_c k}{V_A}\omega - 1 = 0, \tag{30}$$

- whose analytical solution is given by,

$$\omega = \frac{-ik\eta_c \pm \sqrt{4V_A^2 - k^2\eta_C^2}}{2V_A} \tag{31}$$

- From this solution, the condition to have a real part of the wave frequency equal to zero [See Figures c) and d)] is,

$$k = \frac{2V_A}{\eta_c}, \tag{32}$$

- which determines the cut-off wavenumber for Alfvén waves in partially ionized plasmas in the single-fluid MHD approximation

Dispersion Relations and Results

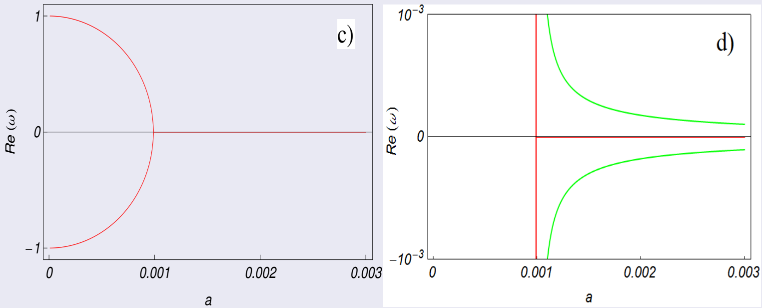

Figure: Real part of the dimensionless wave frequency $Re(\omega)$ versus the dimensionless Alfvén frequency $a = kV_A/\omega_i$ in partially ionized plasmas, where ω_i is the ion gyro-frequency. c) Single-fluid MHD equations without modified Hall current; d) Zoom of the solution in Figure c) (red line) near x axis.

Dispersion Relations and Results

- Again, for fully ionized resistive plasma, $\eta_c = \eta$, we recover the Ferraro & Plumpton (1961) and Kendall & Plumpton (1964) cut-off wavenumber
$$k = \frac{2V_A}{\eta},$$
- Furthermore, if the kinematic viscosity, ν, was considered, we would recover the Chandrasekhar (1961) cut-off wavenumber,
$$k = \frac{2V_A}{\nu - \eta},$$

Modified Alfvén speed

- Finally, from Eq. (31), and in dimensional form, the real part of the Alfvén frequency can be written as,

$$\varpi_r = \pm k V_A \sqrt{1 - \frac{k^2 \eta_C^2}{4 V_A^2}} = \pm k \Gamma_A \tag{33}$$

- with

$$\Gamma_A = V_A \sqrt{1 - \frac{k^2 \eta_C^2}{4 V_A^2}} \tag{34}$$

- representing the modified Alfvén speed for the considered case. When the cut-off wavenumber is attained, the modified Alfvén speed becomes zero
- The following Figure shows the behaviour of the modified Alfvén speed for physical conditions corresponding to photosphere, low chromosphere, high chromosphere, and prominences. In all the cases a cut-off wavenumber appears

Modified Alfvén speed

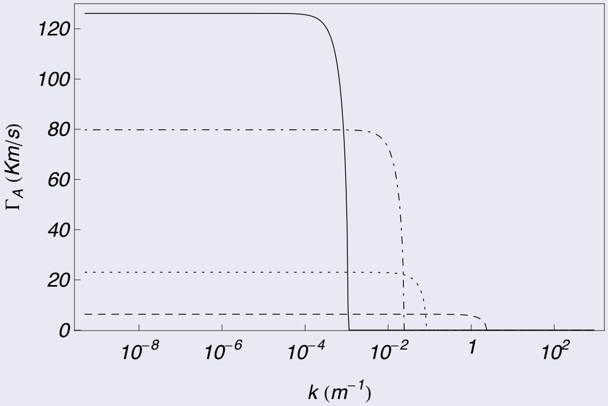

Figure: Modified Alfvén speed, Γ_A, versus wavenumber, k, obtained from Eq. (34). Solid: Quiescent prominence; dash-dotted: High chromosphere; dotted: Low chromosphere; dashed: Photosphere. Physical conditions from FAL1993

Conclusions

- Consequent approximations from multi-fluid to single fluid MHD allow us to find the stage where the cut-off wavenumber for Alfvén waves appears in partially ionized plasmas of the solar atmosphere
- The cut-off wavenumber of Alfvén waves in partially ionized plasmas appears after neglecting the inertial term, and the modified Hall current term in the induction equation
- In conclusion, the cut-off wavenumber of Alfvén waves in single-fluid partially ionized plasma is due to the approximations made when going from multifluid to single fluid equations

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