The Kelvin-Helmholtz instability in weakly ionised flows

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Collaborators

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Why weakly ionised?

Certain regions of the ISM contain mostly neutral material

- Molecular clouds

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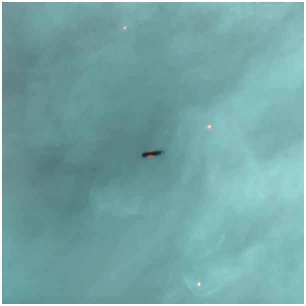

- Accretion disks around YSOs

Weak ionisation (pretty much) implies multifluid effects at *some* length scale

Assumptions

- The bulk flow velocity *is* the neutral velocity
- The majority of collisions experienced by each charged species occur with neutrals
- The charged species' inertia is unimportant
- The charged species' pressure gradient is unimportant

We can derive a generalised Ohm's law for this case.

Outline of derivation of Ohm's law

The momentum equations for the charged species are:

$$\alpha_i \rho_i \left(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}\right) + f_{i1} = 0 \qquad (1)$$

where $i = 2, \ldots, N$. Ignoring mass transfer between the charged species, we can say

$$f_{ij} = \rho_i \rho_j K_{ij} \left(\mathbf{v}_j - \mathbf{v}_i\right) \qquad (2)$$

Outline of derivation of Ohm's law

Moving to the rest frame of the neutral fluid:

$$0 = \alpha_i \rho_i \left(\mathbf{E}' + \mathbf{v}_i' \times \mathbf{B}\right) - \frac{B}{\beta_i}(\alpha_i \rho_i \mathbf{v}_i') \tag{3}$$

Outline of derivation of Ohm's law

After a little algebra:

$$\mathbf{J} = \sigma_{\|}\mathbf{E}'_{\|} + \sigma_{\perp}\mathbf{E}'_{\perp} + \sigma_H(\mathbf{E}' \times \mathbf{b}) \tag{4}$$

where $\mathbf{b} \equiv \frac{\mathbf{B}}{B}$. Hence

$$\mathbf{E}' = r_0\frac{(\mathbf{J}\cdot\mathbf{B})\mathbf{B}}{B^2} + r_1\frac{\mathbf{J}\times\mathbf{B}}{B} + r_2\frac{\mathbf{B}\times(\mathbf{J}\times\mathbf{B})}{B^2} \tag{5}$$

The induction equation

Our induction equation then becomes

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot \{\mathbf{uB} - \mathbf{Bu}\} = \nabla \times \left\{ r_0 \frac{(\mathbf{J} \cdot \mathbf{B})\mathbf{B}}{B^2} + r_1 \frac{\mathbf{J} \times \mathbf{B}}{B} + r_2 \frac{\mathbf{B} \times (\mathbf{J} \times \mathbf{B})}{B^2} \right\} \tag{6}$$

The equations ...

$$
\begin{aligned}
\frac{\partial \rho_i}{\partial t} + \boldsymbol{\nabla} \cdot (\rho_i \boldsymbol{u}_i) &= 0, (1 \le i \le N), \\
\frac{\partial \rho_1 \boldsymbol{u}_1}{\partial t} + \nabla \cdot \left(\rho \boldsymbol{u}_1 \boldsymbol{u}_1 + a^2 \rho \mathbf{I} \right) &= \boldsymbol{J} \times \boldsymbol{B}, \\
\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \cdot (\boldsymbol{u}_1 \boldsymbol{B} - \boldsymbol{B} \boldsymbol{u}_1) &= \nabla \times \left\{ r_0 \frac{(\mathbf{J} \cdot \mathbf{B})\mathbf{B}}{B^2} + r_1 \frac{\mathbf{J} \times \mathbf{B}}{B} + r_2 \frac{\mathbf{B} \times (\mathbf{J}}{B^2} \right. \\
\alpha_i \rho_i \left(\boldsymbol{E} + \boldsymbol{u}_i \times \boldsymbol{B} \right) &= -\rho_i \rho_1 K_{i\,1} (\boldsymbol{u}_1 - \boldsymbol{u}_i), 2 \le i \le N, \\
\nabla \cdot \boldsymbol{B} &= 0, \\
\nabla \times \boldsymbol{B} &= \boldsymbol{J}, \\
\sum_{i=2}^{N} \alpha_i \rho_i &= 0.
\end{aligned}
$$

Where's the problem?

We don't like diffusive terms:

- For explicit algorithms they limit the time-step we can take with each iteration
- In extreme systems the Hall effect limits the time-step to zero.

We don't like implicit algorithms:

- Challenging to make multidimensional
- Challenging to parallelise

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"Diffusion" terms in our induction equation:

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Ambipolar *diffusion* causes a serious stable time-step problem.

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Hall can be a very big problem.

Outline of numerics

Advance the entire system of equations using operator splitting (O'Sullivan & Downes 2006, 2007):

- Advance neutrals using Godunov-type method
- Apply "diffusion terms" using super-time-stepping and the Hall Diffusion Scheme
- Advance charged species densities assuming force balance
- Method of Dedner used to control $\nabla \cdot \boldsymbol{B}$

Method is entirely *explicit*

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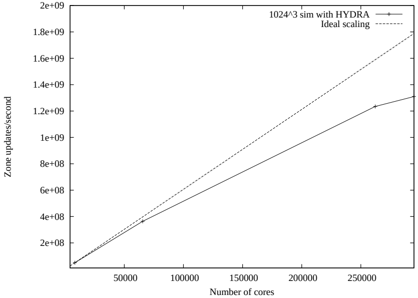
Scaling

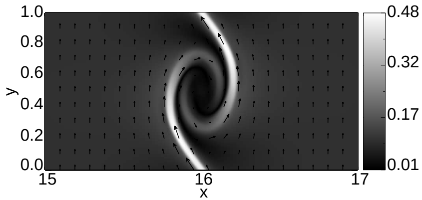


Strong scaling on the JUGENE BG/P system at Juelich (1024^3)

Initial conditions - KH instability

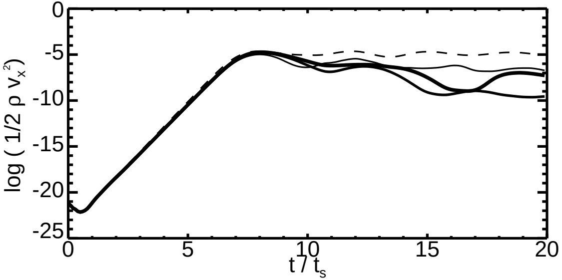
- Isothermal, multifluid MHD: neutrals, electrons and ions.
- Computational domain in (x, y) of $32L \times L$, resolution of 6400×200
- Flow in the y direction, periodic boundaries at high and low y, gradient zero and high and low x
- Ambipolar dominated and Hall dominated flows (magnetic Reynolds numbers in the range of 28 – 280.

Ambipolar dominated KH instability



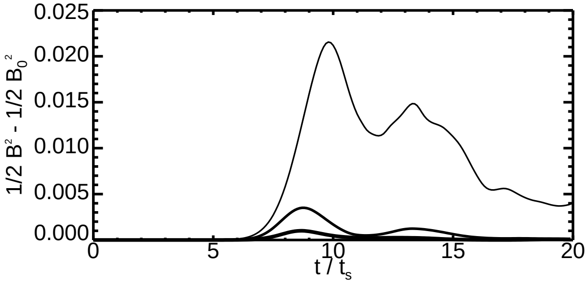
Magnitude (grey-scale) and vector field of the magnetic field for ideal (left panel) and ambipolar dominated (right panel) simulations at onset of saturation

Ambipolar dominated KH instability



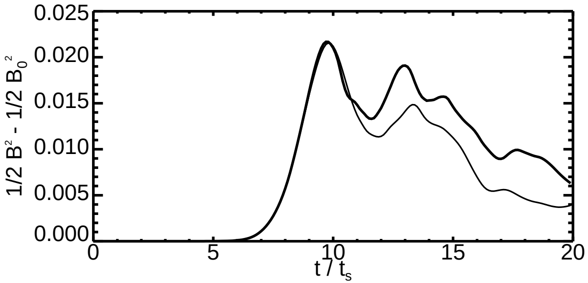
Transverse kinetic energy as a function of time (progressively thicker lines for high ambipolar resistivity)

Ambipolar dominated KH instability



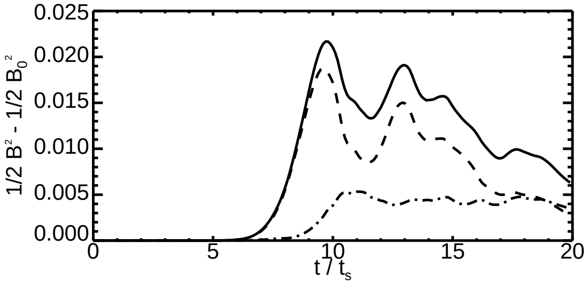
As previous slide, but for perturbed magnetic energy

Hall dominated KH instability



Perturbed magnetic field evolution in Hall dominated, and ideal MHD simulations.

Hall dominated KH instability

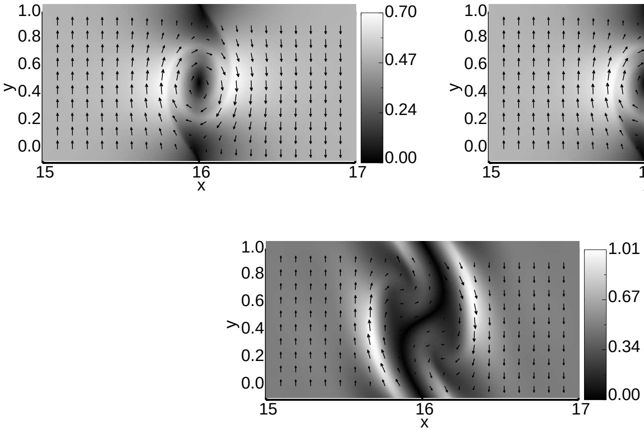


As previous slide, but decomposing magnetic energy into that in the *xy*-plane and that in the *z* direction.

Hall dominated KH instability

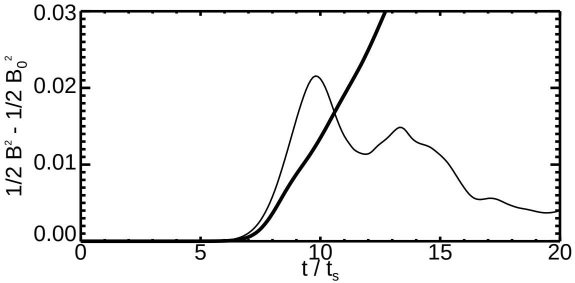
So now let's boost the Hall resistivity even further ...

Hall dominated KH instability



Plots of the magnitude (grey-scale) and vector field of the neutral, ion and electron velocity fields

Hall dominated KH instability



Perturbed magnetic field evolution in Hall dominated, and ideal MHD simulations.

Conclusions

- Ambipolar diffusion dramatically reduces the magnetic energy generated, and marginally increases the peak transverse energy
- The Hall effect leads to a system which does not reach a quasi-steady state
- In extreme situations the Hall effect leads to strong dynamo action