Damping of MHD waves in the solar partially ionized plasmas

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➢ MHD waves on the Sun

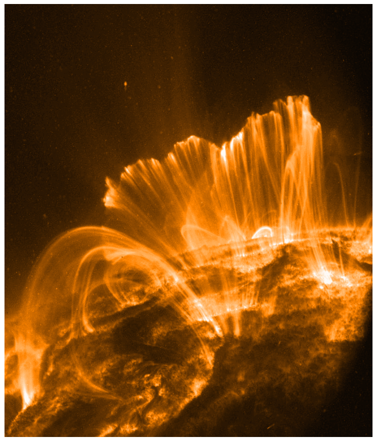
- **Magnetic field plays the key role in the solar activity:**

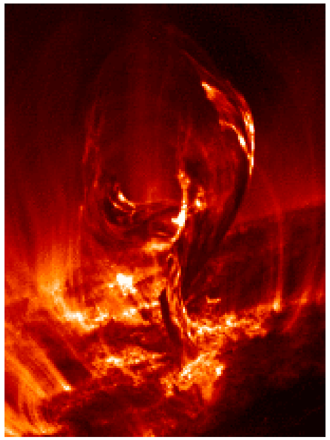
  - **It gives the *origin of solar active regions* and their internal structure**

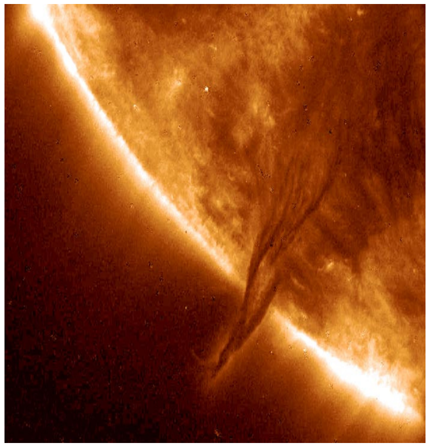
  - **It *controls the dynamics* of solar plasma and appears as an important factor of solar energetic phenomena (flares, CMEs, prominences)**

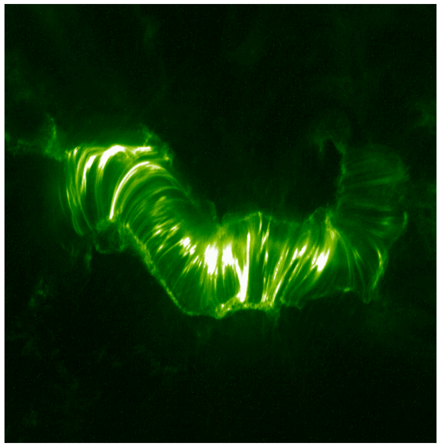
  - **It *structurizes* the solar atmosphere (loops, filaments)**

  - **It *channels the energy* from the convection zone and photosphere towards the upper solar atmosphere (MHD & sound waves**









➢ MHD waves on the Sun

- **MHD waves as a heating source for outer solar atmosphere**

Dissipation mechanism:

- ***collisional*** friction
- ***viscousity***
- ***thermoconductivity***

→

Conversion of damped MHD waves energy into thermal energy, i.e. ***heating***

- **Damping of MHD waves is applied for explanation of**

  - **Non-uniform heating of chromospheric foot-points of magnetic loops and energy deposition in solar plasmas (slow m/s. w.)**
  - **Driver for solar spicules (A.w. and fast m/s. w.)**
  - **Damping of coronal loop oscillations (leakage of A.w.energy through the foot points)**
  - **Damped oscillations of prominences (damped A.w. and m/s w.)**

➢ MHD waves damping

- **Different physical nature of the viscous and frictional damping:**
  - **The forces associated with the viscosity and thermal conductivity have purely kinetic origin and are caused by momentum transfer during the thermal motion of particles**
  - **The collisional friction forces appear due to the average relative motion of the plasma species as a whole**

- **MHD waves damping in a <u>linear</u> approximation**

(approach by Braginskii, S.I., Transport processes in plasma, in: Reviews of plasma phys., 1, 1965)

→ **Calculation of energy decay time using the local heating rates:** Q_{frict}, Q_{visc} (= ½ $\pi_{\alpha\beta} W_{\alpha\beta}$), Q_{therm} (= - $q\nabla T/T_0$), **etc.**

→ **The decay of wave amplitude is described by complex frequency:** $\omega - i\omega\delta$ **where** δ **(<<1) is the logarithmic damping decrement**

→ **The energy** ε **decays as:** $e^{-t/\tau}$, **where** $\tau = (2\,\omega\,\delta)^{-1}$ **is a wave damping time**

→ $2\,\omega\,\delta = (1/\varepsilon)\,T_0\,\theta \;\; = (1/\varepsilon)\,\Sigma Q_i$, **where** θ **is the entropy production rate**

➢ MHD waves damping

- **Collisional friction dissipation ⇒ Joule dissipation:**

Fully ionized plasma

$$\boxed{Q_{\text{frict}} = Q_{\text{Joule}} = \frac{j_{\parallel}^2}{\sigma_{\parallel}} + \frac{j_{\perp}^2}{\sigma_{\perp}}}$$

$\sigma_{\parallel}$ and $\sigma_{\perp}$ are the components of electro-conductivity relative *B*

$\sigma_{\parallel} \approx 2\,\sigma_{\perp}$ in plasma with $Z = 1$

(i.e., $q_i/e = Z$)

Partially ionized plasma

$$\boxed{Q_{\text{frict}} = \frac{j^2}{\sigma} + \frac{1}{\alpha_{\text{n}}}\left(\frac{\xi_{\text{n}}}{c}[\boldsymbol{j}\times\boldsymbol{B}] - \boldsymbol{G}\right)^2}$$

$$\boldsymbol{G} = \xi_{\text{n}}\nabla(p_{\text{e}} + p_{\text{i}}) - \xi_{\text{i}}\nabla p_{\text{n}}$$

$$\alpha_{\text{n}} = m_{\text{e}}n_{\text{e}}\nu'_{\text{en}} + m_{\text{i}}n_{\text{i}}\nu'_{\text{in}}$$

$$\xi_{\text{i}} = \frac{m_{\text{i}}n_{\text{i}}}{m_{\text{n}}n_{\text{n}} + m_{\text{i}}n_{\text{i}}}, \quad \xi_{\text{n}} = \frac{m_{\text{n}}n_{\text{n}}}{m_{\text{n}}n_{\text{n}} + m_{\text{i}}n_{\text{i}}}$$

$$\nu'_{kl} = \frac{m_l}{m_l + m_k}\nu_{kl}\;,\quad k = e,i\;;\quad l = i,n$$

when $\cancel{G}$ *(A.w. & f. ms.w.)*

$$\boxed{Q_{\text{frict}} = \tilde{Q}_{\text{Joule}} = \frac{j_{\parallel}^2}{\sigma} + \frac{j_{\perp}^2}{\sigma_{\text{C}}}} \qquad \boxed{\sigma_{\text{C}} = \frac{\sigma}{1 + \frac{\xi_{\text{n}}^2 B_0^2}{\alpha_{\text{n}} c^2}\sigma}}$$

MHD waves damping

- **Linear damping <u>due to friction</u> (*Braginskii, 1965*):**

Fully ionized plasma | ***Partially ionized plasma***

Alfvén wave (*A.w.*)

$$2\omega\delta^{\text{A.w.}}_{\text{Joule}} \equiv \frac{1}{\tau^{\text{A.w.}}_{\text{Joule}}} = \frac{c^2}{4\pi\sigma_{\|}}k_\perp^2 + \frac{c^2}{4\pi\sigma_\perp}k_\|^2$$

i.e., *G=0*

$$2\omega\tilde{\delta}^{\text{A.w.}}_{\text{Joule}} \equiv \frac{1}{\tilde{\tau}^{\text{A.w.}}_{\text{Joule}}} = \frac{c^2}{4\pi\sigma}k_\perp^2 + \frac{c^2}{4\pi\sigma_{\text{C}}}k_\|^2$$

Fast magnetoacoustic (or magnetosonic) wave (*f. ms.w.*)

$$2\omega\delta^{\text{f.ms.w.}}_{\text{Joule}} \equiv \frac{1}{\tau^{\text{f.ms.w.}}_{\text{Joule}}} = \frac{c^2}{4\pi\sigma_\perp}k^2$$

i.e., $G \sim \xi_i C_s^2/V_A^2 \ll 1$

$$2\omega\tilde{\delta}^{\text{f.ms.w.}}_{\text{Joule}} \equiv \frac{1}{\tilde{\tau}^{\text{f.ms.w.}}_{\text{Joule}}} = \frac{c^2}{4\pi\sigma_{\text{C}}}k^2$$

Acoustic (or sound) wave (*s.w.*) **the case $m_i = m_n$**

$$2\omega\delta^{\text{s.w.}}_{\text{Joule}} \equiv \frac{1}{\tau^{\text{s.w.}}_{\text{Joule}}} = \frac{c^2}{4\pi\sigma_\perp}k_\perp^2\frac{C_{\text{s}}^2}{V_{\text{A}}^2}$$

$$2\omega\tilde{\delta}^{\text{s.w.}}_{\text{frict}} \equiv \frac{1}{\tilde{\tau}^{\text{s.w.}}_{\text{frict}}} = \frac{c^2}{4\pi\sigma}\frac{C_s^2}{V_A^2}k_\perp^2 + \frac{\xi_n^2 C_s^2\rho_0}{\alpha_n}\left(k_\|^2\frac{n_e^2}{n_0^2} + k_\perp^2\frac{(n_i+n_n)^2}{n_0^2}\right)$$

MHD waves damping

- **Linear damping due to viscosity (*Braginskii, 1965*):**

The same expressions as in fully ionized plasma,

→ **Alfvén wave (*A.w.*)** $$2\omega\delta_{\rm visc}^{\rm A.w.} \equiv \frac{1}{\tau_{\rm visc}^{\rm A.w.}} = \frac{1}{\rho_0}\left(\eta_1 k_\perp^2 + \eta_2 k_\parallel^2\right)$$

→ **Fast magnetosonic wave (*f. ms.w.*)** $$2\omega\delta_{\rm visc}^{\rm f.ms.w.} \equiv \frac{1}{\tau_{\rm visc}^{\rm f.ms.w.}} = \frac{1}{\rho_0}\left[\left(\frac{\eta_0}{3} + \eta_1\right)k_\perp^2 + \eta_2 k_\parallel^2\right]$$

→ **Acoustic (or sound) wave (*s.w.*)** $$2\omega\delta_{\rm visc}^{\rm s.w.} \equiv \frac{1}{\tau_{\rm visc}^{\rm s.w.}} = \frac{1}{\rho_0}\left(\frac{4}{3}\eta_0 k_\parallel^2 + \eta_2 k_\perp^2\right)$$

but with ρ_0 and viscosity coefficients, η_0, η_1, η_2, modified to include neutrals

- **In weakly ionized plasma $\eta_{0,1,2} \sim n_n T \tau_n$ (i.e., isotropy), $\tau_n = (\nu_{ni} + \nu_{nn})^{-1}$**
- **for $n_i/n_n \geq 1$ ion viscosity still dominates, but $\tau_i = (\nu_{ii} + \nu_{in})^{-1}$**

$$\eta_0 = 0.96 n_i T_i \tau_i \quad \eta_2 = n_i T_i \tau_i \frac{\left(\frac{6}{5}(\omega_i\tau_i)^2 + 2.23\right)}{(\omega_i\tau_i)^4 + 4.03(\omega_i\tau_i)^2 + 2.33} \quad \eta_1 = \eta_2\ (\omega i \rightarrow 2\omega i)$$

➢ MHD waves damping

- **Linear damping <u>due to thermal conductivity</u> (*Braginskii, 1965*):**

The same expressions as in fully ionized plasma,

→ **Alfvén wave (*A.w.*)** No osc. of ρ and T → no thermal cond. damping

→ **Fast magnetosonic wave (*f. ms.w.*)** $$2\omega\delta^{\text{f.ms.w.}}_{\text{therm}} \equiv \frac{1}{\tau^{\text{f.ms.w.}}_{\text{therm}}} = \frac{(\gamma-1)^2 T k_\perp^2}{\rho_0 V_A^2 k^2}\left(æ_\parallel k_\parallel^2 + æ_\perp k_\perp^2\right)$$

→ **Acoustic (or sound) wave (*s.w.*)** $$2\omega\delta^{\text{s.w.}}_{\text{therm}} \equiv \frac{1}{\tau^{\text{s.w.}}_{\text{therm}}} = \frac{(\gamma-1)^2 T}{\rho_0 C_s^2}\left(æ_\parallel k_\parallel^2 + æ_\perp k_\perp^2\right)$$

but with ρ_0 and therm.cond. coefficients $æ_\parallel$, $æ_\perp$ modified to include neutrals

- **In weakly ionized plasma ϰ ~ $n_n T \tau_n / m_n$ (i.e., isotropy), $\tau_n = (\nu_{ni} + \nu_{nn})^{-1}$**
- **for $n_i/n_n \geq 1$ ion effects still dominate, but $\tau_i = (\nu_{ii} + \nu_{in})^{-1}$**
- **therm.cond along B is mainly due to electrons, but $\tau_e = (\nu_{ei} + \nu_{en})^{-1}$**

$$æ_\perp = \frac{n_i T \tau_i}{m_i}\frac{2(\omega_i\tau_i)^2 + 2.645}{(\omega_i\tau_i)^4 + 2.70(\omega_i\tau_i)^2 + 0.677}$$

$$æ_\parallel = \frac{n_e T \tau_e}{m_e}\gamma_0, \quad \gamma_0$$

➢ Application to the Sun

- **Specifics of the case:**

→ **Longitudinal propagation of waves:** $k_{\parallel} \neq 0$ **and** $k_{\perp} = 0$

(*m. flux tubes in photosphere/chromosphere serve as a wave-guide)*

→ **Transverse propagation of waves:** $k_{\perp} \neq 0$ **and** $k_{\parallel} = 0$

(*m.of interest for waves in prominences)*

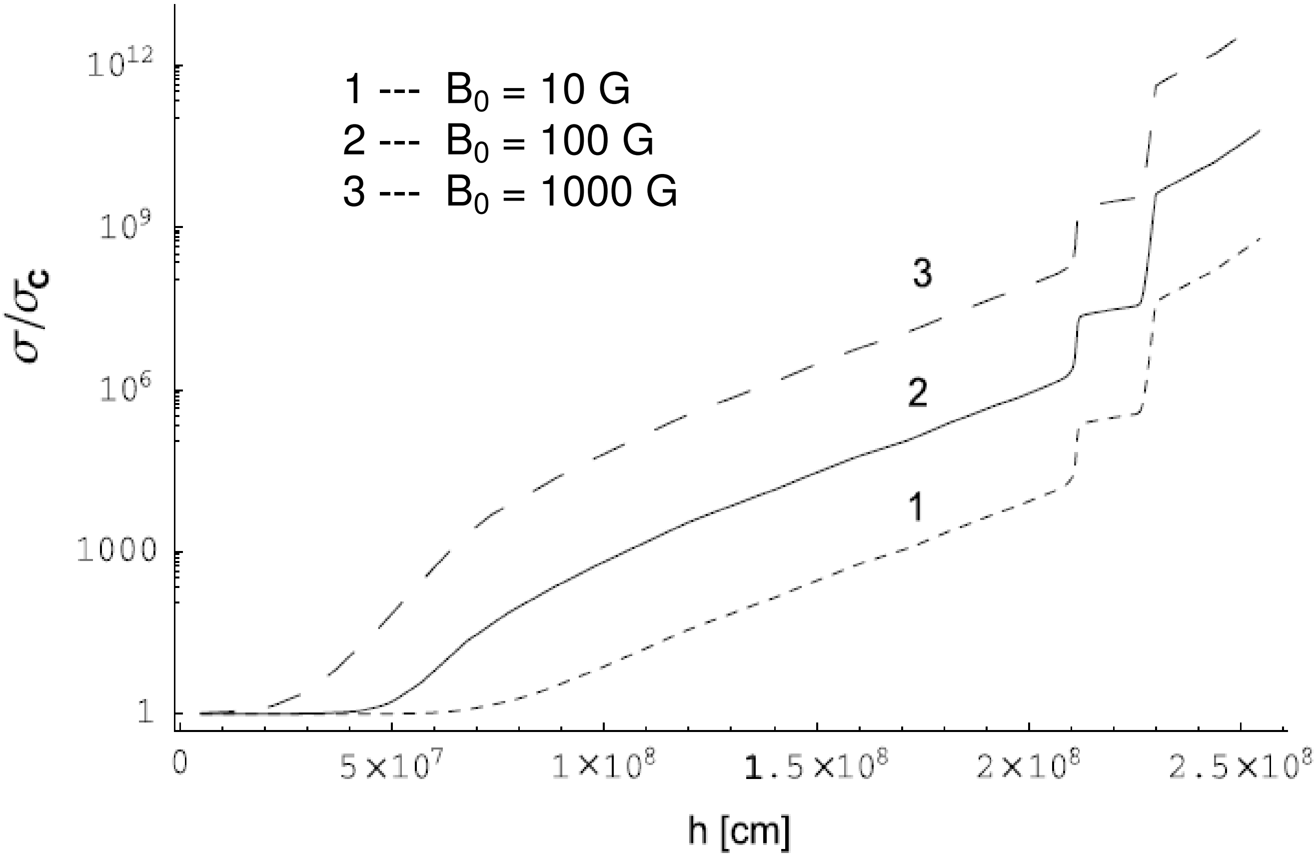
→ **In the case** $k_{\parallel} \neq 0$; $k_{\perp} = 0$ *A.w.* **&** *f.ms.w.* **are** ***equally damped*** **due to** **viscous, as well as due to collision dissipation**

→ **In the partially ionized plasma for collision damping of longitudinal** ($k_{\parallel} \neq 0$, $k_{\perp} = 0$) *A.w.* **&** *f.ms.* **waves** $\sigma \gg \sigma_C$

$$\frac{\sigma}{\sigma_{\mathrm{C}}} = 1 + \xi_{\mathrm{n}}^2 \frac{B_0^2 \sigma}{\alpha_{\mathrm{n}} c^2} \approx 1 + \xi_{\mathrm{n}}^2 \frac{\omega_{\mathrm{e}} \omega_{\mathrm{i}}}{\max\{\nu'_{\mathrm{ei}}, \nu'_{\mathrm{en}}\} \nu'_{\mathrm{in}}} \gg 1 .$$

➢ Application to the Sun

- **Specifics of the case:**



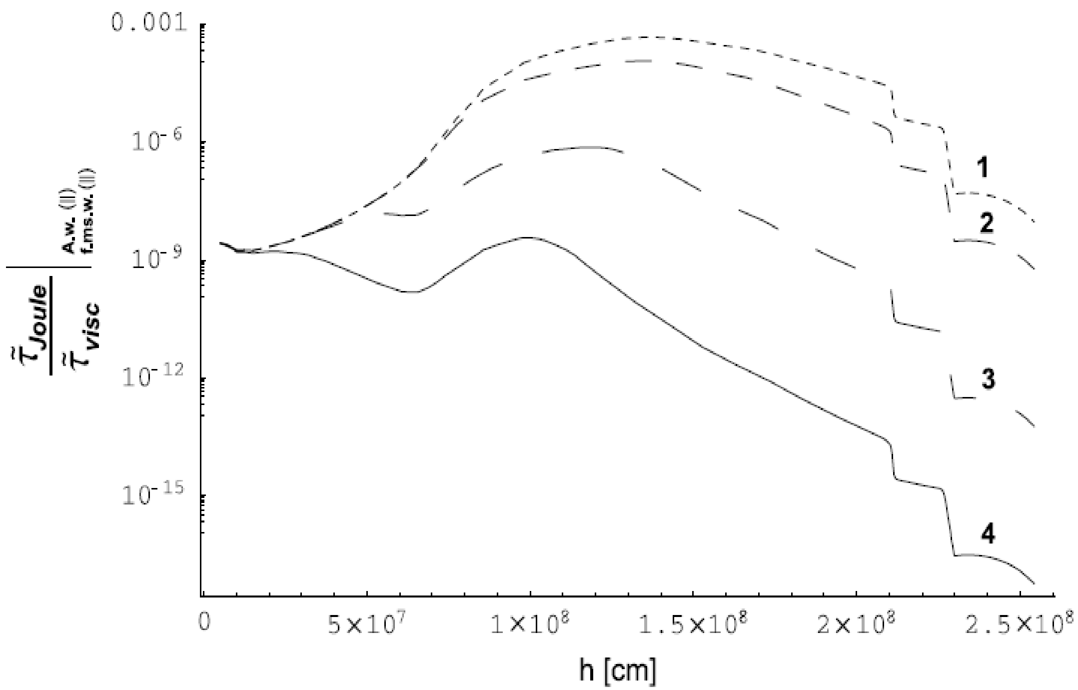
Variation of σ/σ_C with height for the quiet Sun model "VAL C" (*Vernazza, J.E., Avrett, E.H., Loeser, R., ApJ Suppl, 45, 635, 1981*)

in a strong enough m.field $\sigma >> \sigma_C$ → stronger collision damp. in p.i.p.

➢ Application to the Sun

- **Low solar atmosphere:**
  - ♦ **Longitudinally propagating** ($k_{\parallel} \neq 0$ *and* $k_{\perp} = 0$) ***A.w.*** **&** ***f.ms.w.***

→ **collisional damping vs. viscous damping** $\left.\dfrac{\tau_{\text{Joule}}}{\tau_{\text{visc}}}\right|_{\substack{\text{A.w.}(\parallel)\\ \text{f.ms.w.}(\parallel)}} = \dfrac{4\pi\eta_2\sigma_C}{c^2\rho_0}$



1 --- B_0 = 5 G
2 --- B_0 = 10 G
3 --- B_0 = 100 G
4 --- B_0 = 1000 G

Collision damping of A.w. & f.ms.w. dominates in photosphere and chromosphere

→ **no thermal conductivity damping of** *A.w.* **&** *f.ms.w.* **for** $k_{\parallel} \neq 0$; $k_{\perp} = 0$

➢ Application to the Sun

- **Low solar atmosphere:**

  ♦ **Longitudinally propagating (*k*⌨ ⏲ *0 and k*$_{\gamma}$ *= 0*) *s.w.***

  → **(1) collisional damping vs. viscous damping:**

$$\left.\frac{\tau_{\rm frict}}{\tau_{\rm visc}}\right|_{\rm s.w.(\|)} = \frac{4}{3}\,\frac{\eta_0\alpha_n}{\xi_n^2 C_s^2 \rho_0^2}\,\frac{n_0^2}{n_e^2}$$

  → **(2) collisional damping vs. thermal cond. damping:**

$$\left.\frac{\tau_{\rm frict}}{\tau_{\rm therm}}\right|_{\rm s.w.(\|)} = \frac{(\gamma-1)^2}{\gamma^2}\,\frac{\alpha_n æ_{\|}}{\xi_n^2 T n_e^2}$$

  → **(3) viscous damping vs. therm. cond. damping:**

$$\left.\frac{\tau_{\rm visc}}{\tau_{\rm therm}}\right|_{\rm s.w.(\|)} = \frac{3(\gamma-1)^2}{4}\,\frac{T æ_{\|}}{C_s^2 \eta_0}$$

  ♦ **Coll. & visc. damping of s.w. in the chromosphere are similar, with slight domination of the coll. damp.**

  ♦ **Therm. cond. damping is more efficient**

➢ Application to the Sun

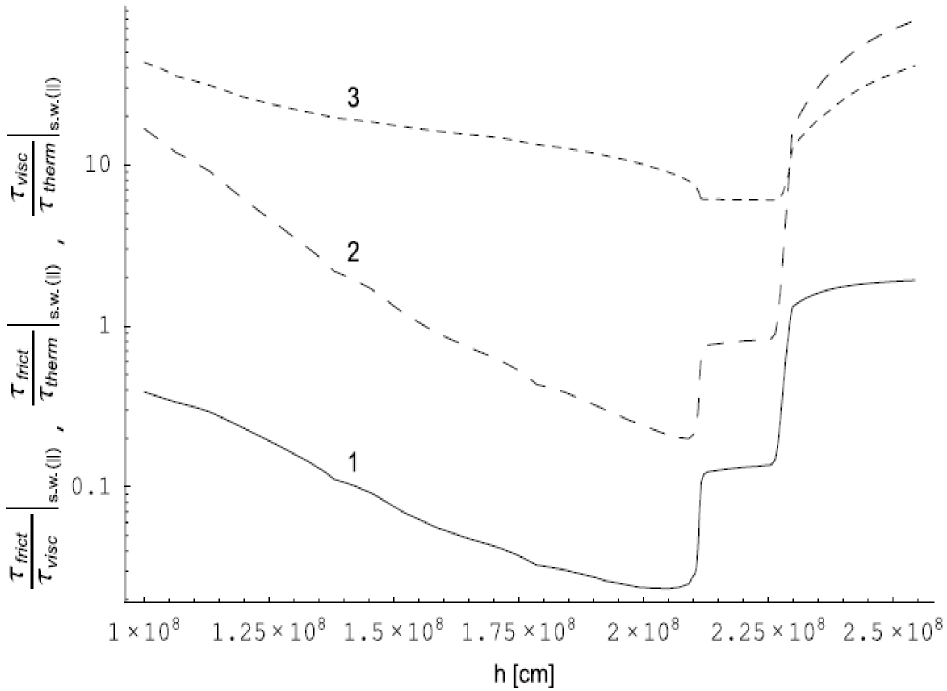
- **Low solar atmosphere:**

♦ **Longitudinally propagating ($k_{\parallel} \neq 0$ and $k_{\gamma} = 0$) *s.w.***

→ **(1) collisional damping vs. viscous damping:**

→ **(2) collisional damping vs. thermal cond. damping:**

→ **(3) viscous damping vs. therm. cond. damping:**



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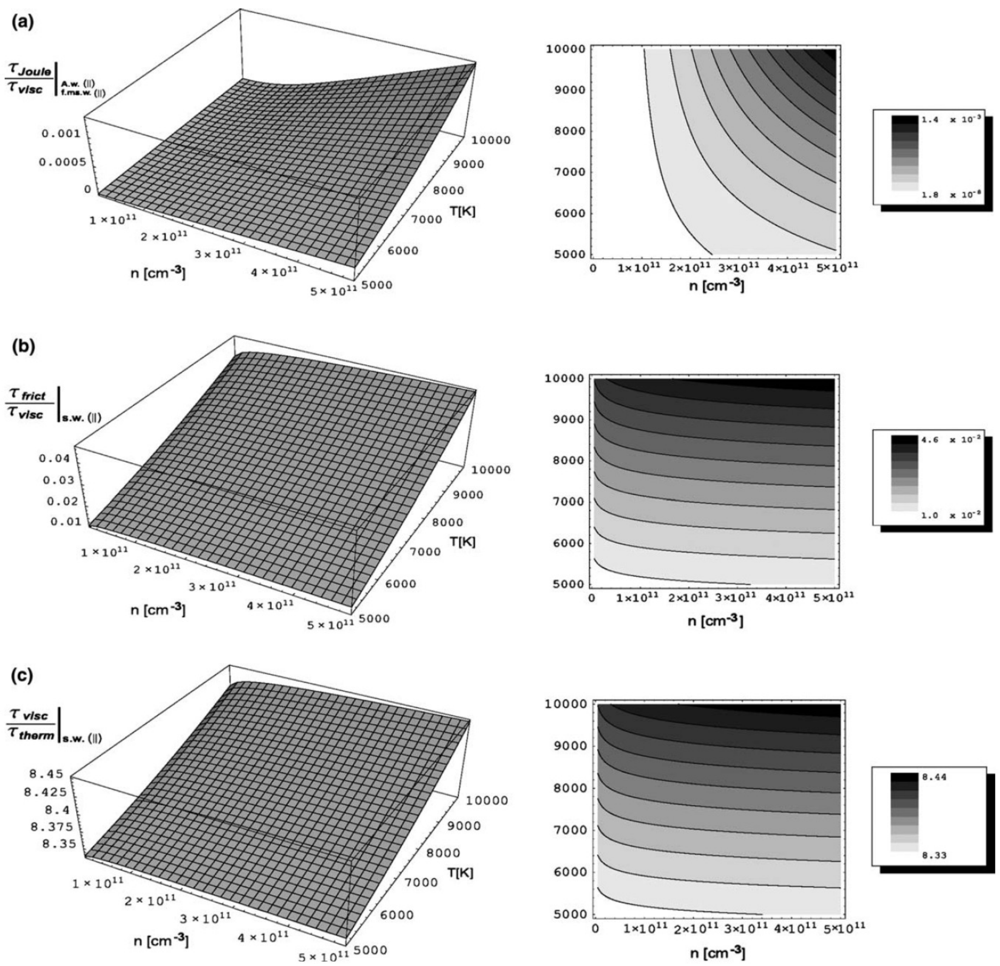
➢ Application to the Sun

- **Solar prominences:** $T=(6...10)\cdot 10^3$ K; $n=(1...50)\cdot 10^{10} cm^{-3}$; $n_n/n=0.05...1$, $B_0 \sim 10$ G
  - **Longitudinally propagating** (k ≠ 0 and $k_{\gamma} = 0$) *A.w.*, *f.ms.w.* & *s.w.*



Calculations with
$B_0=10G$, $n_n/n=1$

- ♦ **for longitudinally prop.** *A.w.*, *f.ms.w*, **and** *s.w.* **the coll.damp. dominates the visc. & therm.cond.damp.**
- ♦ **for** *s.w.* **the therm.cond. damping dominates the viscosity damping**

➢ Application to the Sun

- **Solar prominences:** $T=(6...10)\times 10^3$ K; $n=(1...50)\times 10^{10}\,cm^{-3}$; $n_n/n=0.05...1$, $B_0 \sim 10$ G
  - **Transverse propagating** ($k_{\gamma} \neq 0$ *and* $k_{\|} = 0$) *f.ms.w.* :

→ **(1) collisional damping vs. viscous damping:**

$$\left.\frac{\tau_{\rm Joule}}{\tau_{\rm visc}}\right|_{\rm f.ms.w.(\perp)} = \frac{4\pi(\eta_0/3+\eta_1)\sigma_{\rm C}}{c^2\rho_0} = \frac{\eta_0/3+\eta_1}{\eta_2}\left.\frac{\tau_{\rm Joule}}{\tau_{\rm visc}}\right|_{\rm f.ms.w.(\|)}$$

> 1

→ **(2) collisional damping vs. thermal cond. damping:**

$$\left.\frac{\tau_{\rm Joule}}{\tau_{\rm therm}}\right|_{\rm f.ms.w.(\perp)} = \frac{4\pi\sigma_{\rm C}(\gamma-1)^2 T æ_{\perp}}{c^2\rho_0 V_A^2}$$

→ **(3) viscous damping vs. therm. cond. damping:**

$$\left.\frac{\tau_{\rm visc}}{\tau_{\rm therm}}\right|_{\rm f.ms.w.(\perp)} = \frac{(\gamma-1)^2 T æ_{\perp}}{V_A^2(\eta_0/3+\eta_1)}$$

➢ Application to the Sun

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  - **Transverse propagating** ($k_{\parallel} \neq 0$ and $k_{\perp} = 0$) *f.ms.w.* **:**

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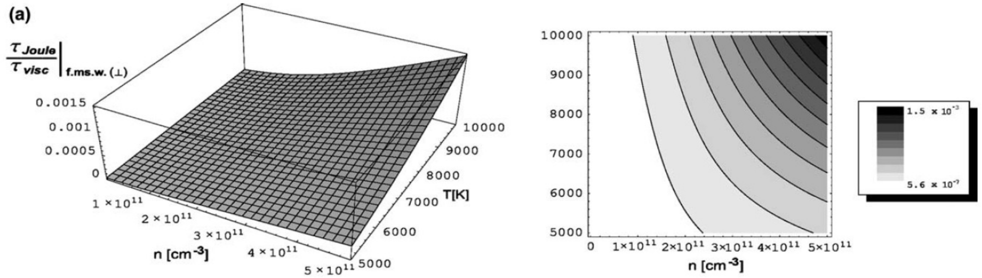
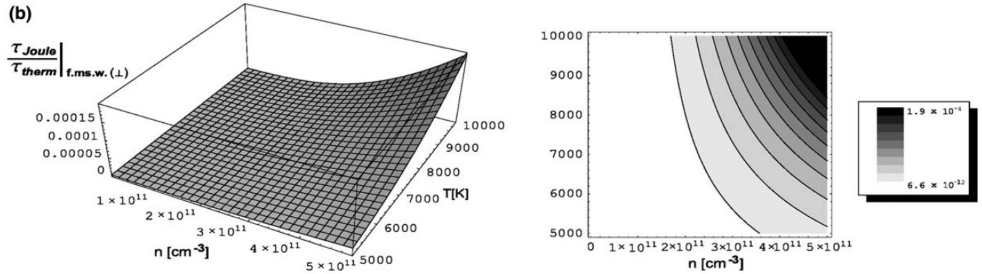


→ **(2) collisional damping vs. thermal cond. damping:**



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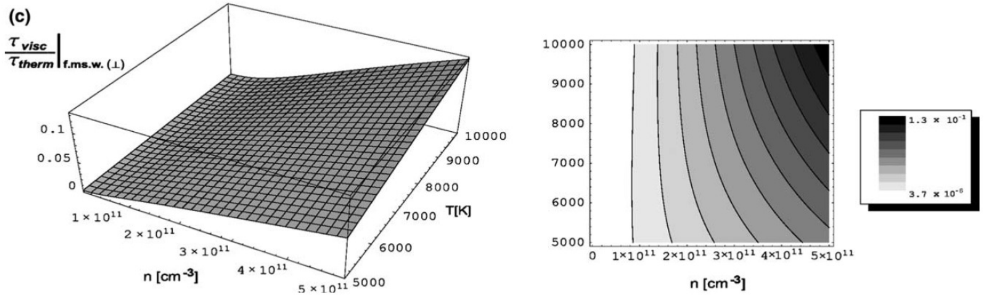

➢ Application to the Sun

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  - ♦ **Transverse propagating** ($k_\perp \neq 0$ *and* $k_\parallel = 0$) *s.w.* **:**

→ **(1) collisional damping vs. viscous damping:**

$$\frac{\tau_{\text{frict}}}{\tau_{\text{visc}}}\bigg|_{\text{s.w.}(\perp)} \approx \frac{\eta_2 \alpha_n}{\xi_n^2 C_s^2 \rho_0^2}\,\frac{n_0^2}{(n+n_n)^2} = \frac{3}{4}\frac{\eta_2}{\eta_0}\left(1+\frac{n_n}{n}\right)^{-2}\frac{\tau_{\text{frict}}}{\tau_{\text{visc}}}\bigg|_{\text{s.w.}(\parallel)}$$

< 1

→ **(2) collisional damping vs. thermal cond. damping:**

$$\frac{\tau_{\text{frict}}}{\tau_{\text{therm}}}\bigg|_{\text{s.w.}(\perp)} \approx \frac{(\gamma-1)^2}{\gamma^2}\,\frac{\alpha_n \ae_\perp}{\xi_n^2 T (n+n_n)^2} = \frac{1}{(1+n_n/n)}\frac{\ae_\perp}{\ae_\parallel}\frac{\tau_{\text{frict}}}{\tau_{\text{therm}}}\bigg|_{s.w.(\parallel)}$$

< 1

→ **(3) viscous damping vs. therm. cond. damping:**

$$\frac{\tau_{\text{visc}}}{\tau_{\text{therm}}}\bigg|_{\text{s.w.}(\perp)} = \frac{(\gamma-1)^2 T \ae_\perp}{C_s^2 \eta_2} = \frac{4}{3}\frac{\eta_0}{\eta_2}\frac{\ae_\perp}{\ae_\parallel}\frac{\tau_{\text{visc}}}{\tau_{\text{therm}}}\bigg|_{\text{s.w.}(\parallel)}$$

< 1

➢ Application to the Sun

- **Solar prominences:** $T=(6...10) \times 10^3$ K; $n=(1...50) \times 10^{10}$ cm^{-3}; $n_n/n=0.05...1$, $B_0 \sim 10$ G

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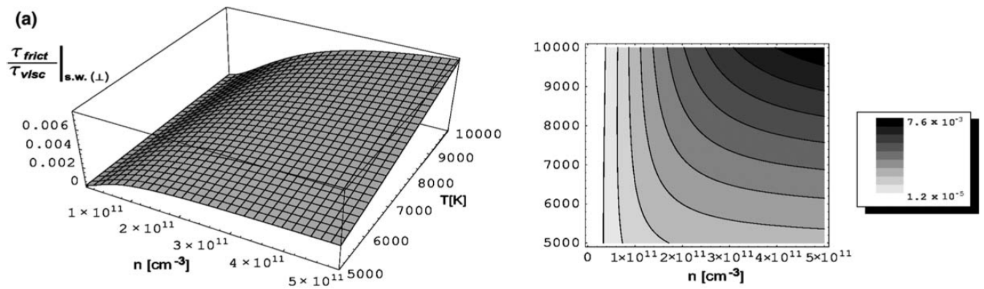
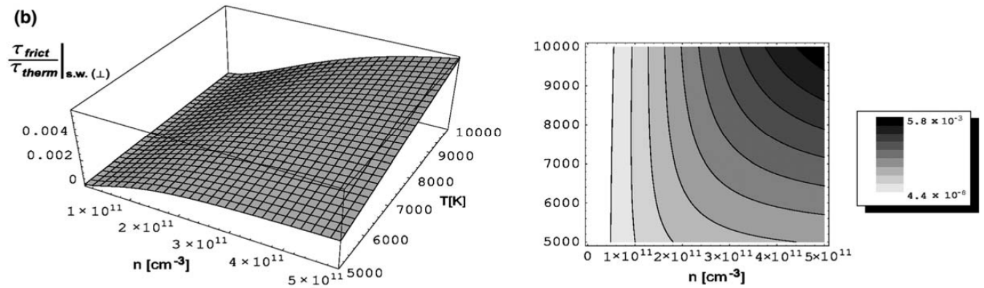


→ **(2) collisional damping vs. thermal cond. damping:**



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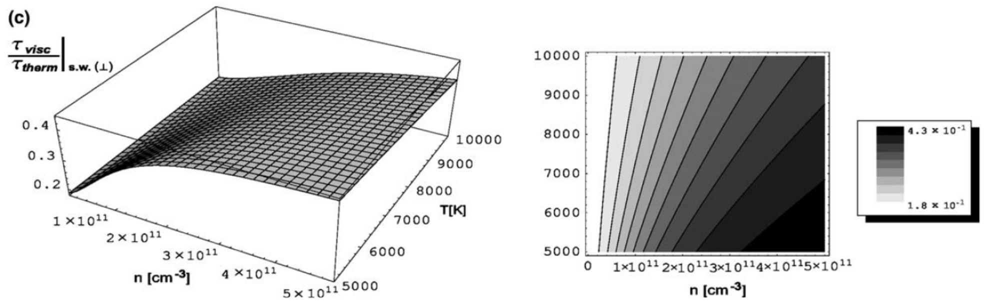

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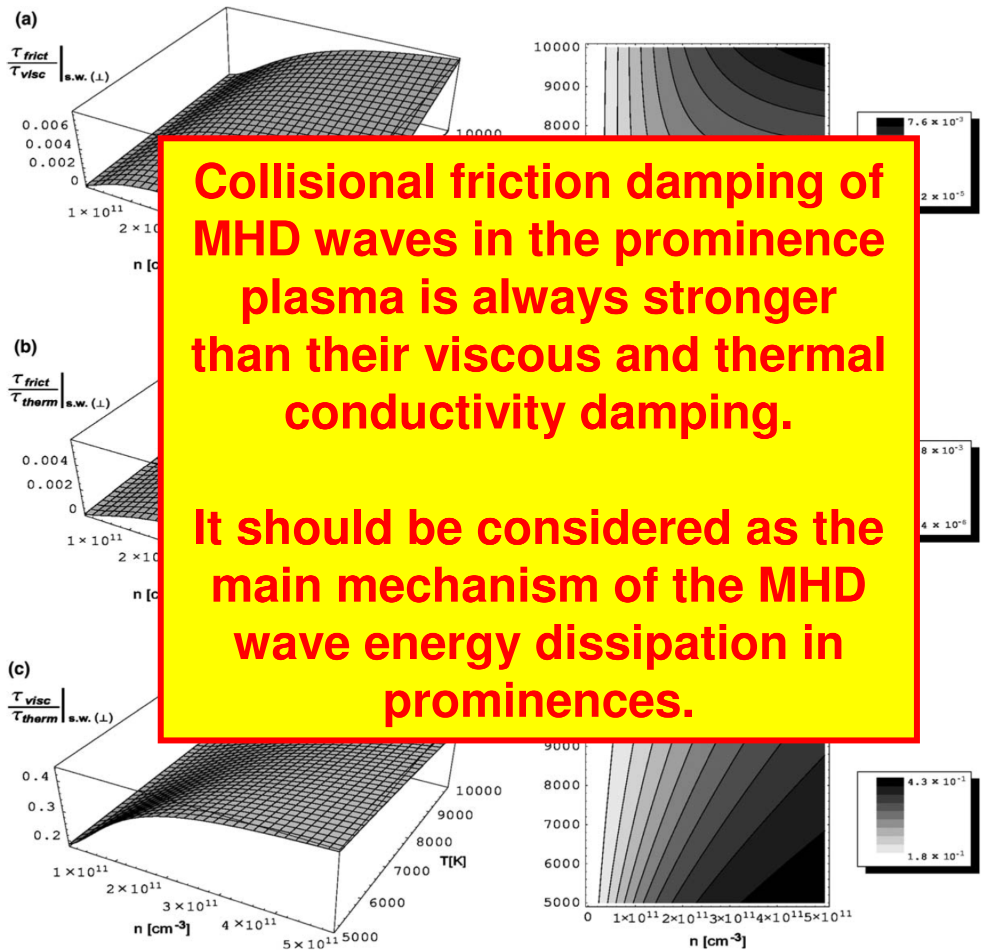

Collisional friction damping of MHD waves in the prominence plasma is always stronger than their viscous and thermal conductivity damping.

It should be considered as the main mechanism of the MHD wave energy dissipation in prominences.

➢ Conclusion

- The collisional friction damping of MHD waves in the solar partially ionized plasmas is usually more important than the viscous and thermal conductivity damping.

- At the same time viscous damping of acoustic waves in some cases (long.prop. in prominences and low atmosphere) can be less efficient then their damping due to the thermal conductivity effects.

- In the middle chromosphere all damping mechanisms are approximately of the same efficiency.

- The expressions used above are valid only if damping decrements $\delta \ll 1$ (linear approximation). Thus, the performed analysis is correct only for waves with frequency $f = \omega/(2\pi) \ll f_c$.

- Depending on plasma parameters, dissipation mechanism, particular MHD mode, the critical frequency f_c varies from 0.1 Hz till 10^6 Hz.