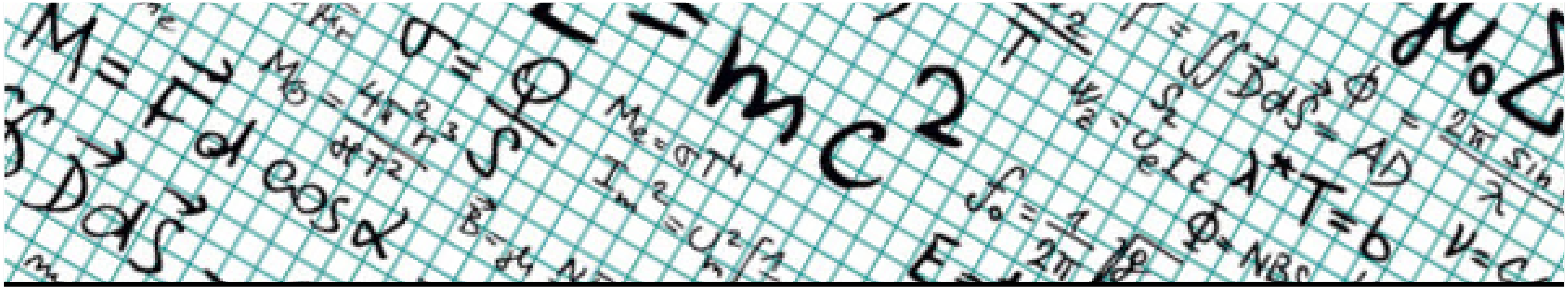

Overview of equations and assumptions

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SPIA
Solar Partially Ionized Atmosphere

http://www.iac.es/proyecto/spia/

Degree of Ionization in VAL-C model

STRUCTURE OF THE SOLAR CHROMOSPHERE. III. MODELS OF THE EUV BRIGHTNESS COMPONENTS OF THE QUIET SUN

J. E. VERNAZZA[1], E. H. AVRETT, AND R. LOESER
Harvard-Smithsonian Center for Astrophysics

	h (km)	m (g cm^{-2})	τ_{500}	T (K)	V (km s^{-1})	n_H (cm^{-3})	n_e (cm^{-3})	P_{total} (dyn cm^{-2})	$\frac{P_{gas}}{P_{total}}$	σ (g cm^{-3})
Chromosphere										
6	2271	5.413-06	5.234-08	32000	9.71	1.378+10	1.498+10	1.483-01	.8976	3.222-14
7	2267	5.427-06	5.657-08	28000	9.70	1.567+10	1.677+10	1.487-01	.8840	3.665-14
8	2263	5.443-06	6.124-08	25500	9.68	1.718+10	1.812+10	1.491-01	.8738	4.017-14
9	2255	5.476-06	7.110-08	24500	9.64	1.797+10	1.881+10	1.500-01	.8698	4.203-14
10	2230	5.583-06	1.030-07	24200	9.49	1.862+10	1.943+10	1.530-01	.8718	4.355-14
Photosphere										
41	555	3.270-02	1.456-04	4230	.70	1.382+15	1.733+11	8.958+02	.9912	3.232-09
42	515	4.878-02	3.014-04	4170	.60	2.096+15	2.495+11	1.336+03	.9934	4.902-09
43	450	9.378-02	1.017-03	4220	.53	3.989+15	4.516+11	2.569+03	.9949	9.327-09
44	350	2.481-01	5.626-03	4465	.52	9.979+15	1.110+12	6.798+03	.9954	2.334-08
45	250	6.172-01	2.670-02	4780	.63	2.315+16	2.674+12	1.691+04	.9936	5.413-08

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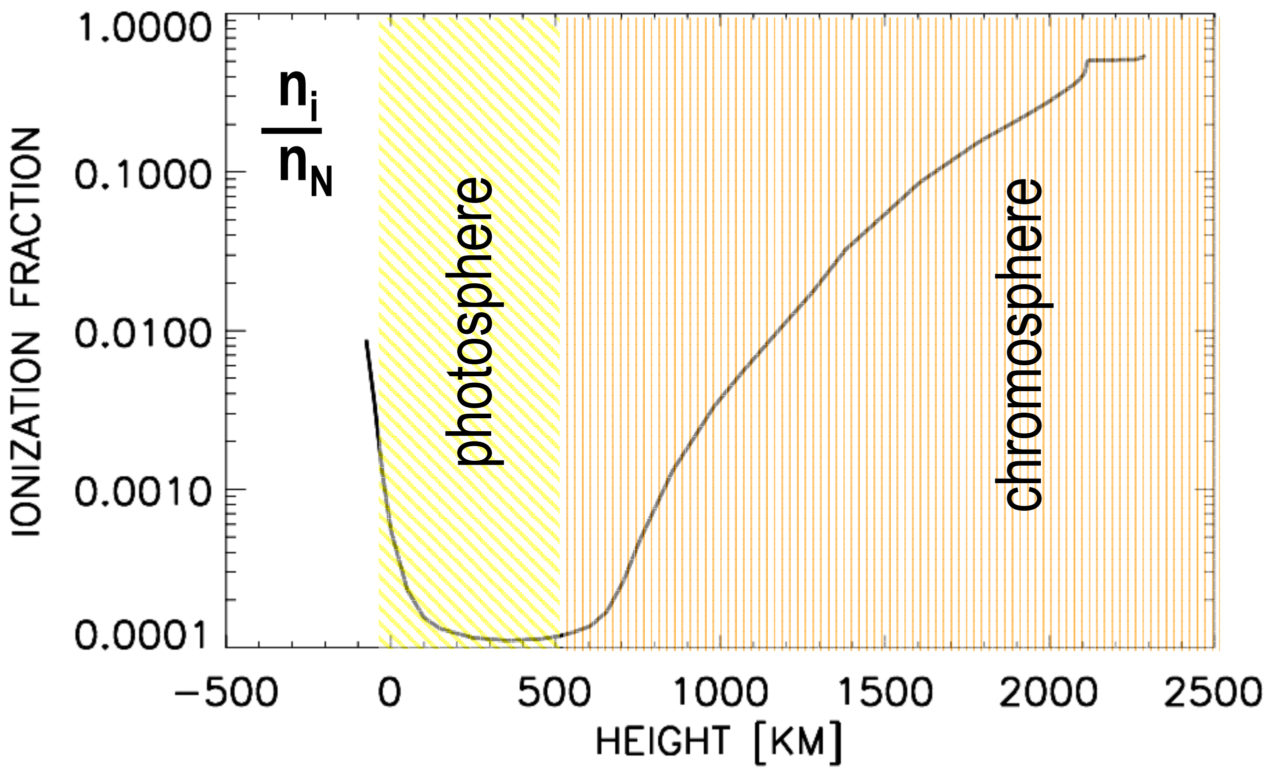
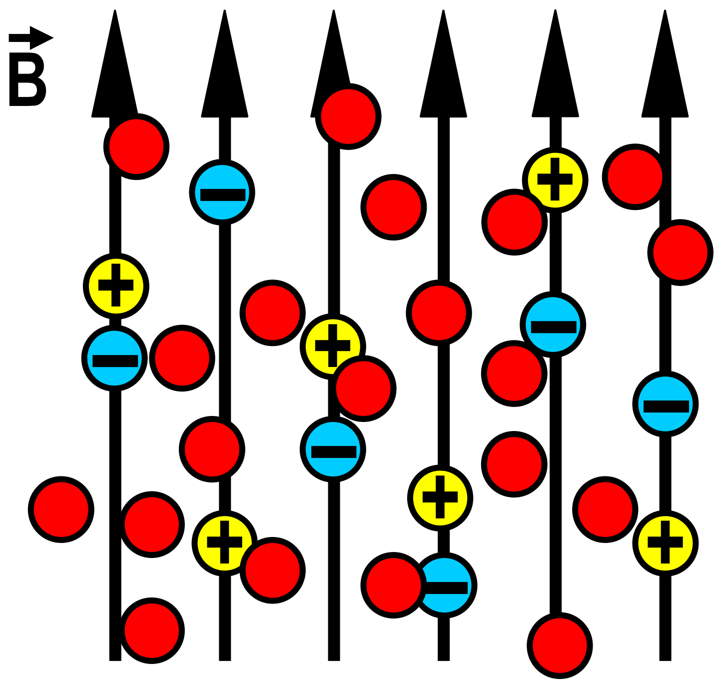
Degree of Ionization in VAL-C model



Multi-fluid approximation



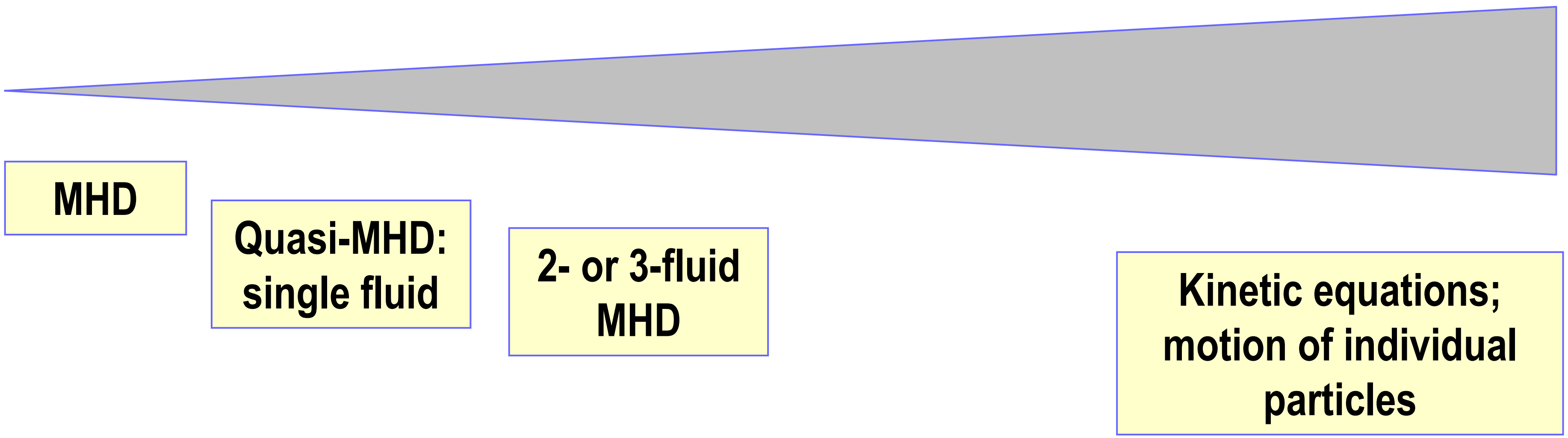
Relative motions between electrons, ions and neutrals (Hall effect and Ambipolar diffusion) produce additional important sources of heating.

Additional diffusitivy due to collisions between ions and neutrals is 5-6 orders of magnitude larger than the usual Coulomb diffusivity (Reynolds number around 1-10).

Characteristic scales in a partially ionized plasma depend on the degree of ionization reaching 10 km and 1 sec for the solar atmosphere.

Important for magnetized photosphere and chromosphere

How detailed?



Depending on scales of phenomena to study, and number of particles.

Events on scales below 10 km & 1 sec and chromospheric density & neutral fraction probably require multi-fluid approach.

Equations for individual species

Mass conservation

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla(\rho_\alpha \vec{u}_\alpha) = S_\alpha \quad \begin{cases} \alpha = e & \text{electrons} \\ \alpha = i & \text{ions} \\ \alpha = n & \text{neutral hydrogen} \end{cases}$$

Momentum conservation

$$\rho_\alpha \frac{D\vec{u}_\alpha}{Dt} = q_\alpha n_\alpha (\vec{E} + \vec{u}_\alpha \times \vec{B}) + \rho_\alpha \vec{g} - \nabla \hat{\mathbf{p}}_\alpha + \vec{R}_\alpha - \vec{u}_\alpha S_\alpha$$

Energy conservation

$$\frac{3}{2}\frac{Dp_\alpha}{Dt} + \frac{3}{2} p_\alpha (\vec{\nabla} \vec{u}_\alpha) + (\hat{\mathbf{p}}_\alpha \vec{\nabla}) \vec{u}_\alpha + \vec{\nabla} \vec{q}_\alpha = M_\alpha - \vec{u}_\alpha \vec{R}_\alpha + \frac{1}{2} u_\alpha^2 S_\alpha$$

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Combined quasi-MHD equations

electrons +ions +neutral hydrogen

Mass conservation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\left(\rho \vec{u}\right) = S$$

Momentum conservation

$$\rho \frac{D\vec{u}}{Dt} = \vec{J} \times \vec{B} + \rho \vec{g} - \vec{\nabla}\hat{\mathbf{p}} - \vec{u} S$$

Energy conservation

$$\frac{3}{2}\frac{Dp}{Dt} + \frac{3}{2} p \vec{\nabla}\vec{u} + \hat{\mathbf{p}}\vec{\nabla}\vec{u} + \vec{\nabla}\vec{q} = \vec{J}\vec{E}^{*} + M + \frac{1}{2}u^2 S$$

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Definitions

see Bittencourt (1986)

$$\vec{u} = \frac{\rho_n \vec{u}_n + \rho_i \vec{u}_i + \rho_e \vec{u}_e}{\rho}$$

Center of mass velocity

$$\hat{\mathbf{p}} = \sum_{\alpha=n,i,e} \hat{\mathbf{p}}_\alpha + \sum_{\alpha=n,i,e} \rho_\alpha \vec{w}_\alpha \otimes \vec{w}_\alpha$$

Kinetic pressure tensor

$$\vec{w}_\alpha \quad = \quad \vec{u}_\alpha - \vec{u}$$

Diffusion velocity

Combined quasi-MHD equations

electrons +ions +neutral hydrogen

Mass conservation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\left(\rho \vec{u}\right) = \cancel{S}$$

Momentum conservation

$$\rho \frac{D\vec{u}}{Dt} = \vec{J} \times \vec{B} + \rho \vec{g} - \vec{\nabla}\hat{\mathbf{p}} - \cancel{\vec{u}S}$$

$$\boxed{\hat{p} = p}$$

Energy conservation

$$\frac{3}{2}\frac{Dp}{Dt} + \frac{3}{2}p\vec{\nabla}\vec{u} + \hat{\mathbf{p}}\vec{\nabla}\vec{u} + \cancel{\vec{\nabla}\vec{q}} = \vec{J}\vec{E}^{*} + \cancel{M} + \cancel{\frac{1}{2}u^2 S}$$

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Combined quasi-MHD equations

electrons +ions +neutral hydrogen

Mass conservation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\left(\rho \vec{u}\right) = 0$$

Momentum conservation

$$\rho \frac{D\vec{u}}{Dt} = \vec{J} \times \vec{B} + \rho \vec{g} - \vec{\nabla} p$$

Energy conservation

$$\frac{3}{2}\frac{Dp}{Dt} + \frac{5}{2} p \vec{\nabla} \vec{u} = \vec{J}\vec{E}^{*}$$

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Combined quasi-MHD equations

electrons +ions +neutral hydrogen

Mass conservation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\left(\rho \vec{u}\right) = 0$$

Momentum conservation

$$\rho \frac{D\vec{u}}{Dt} = \vec{J} \times \vec{B} + \rho \vec{g} - \vec{\nabla} p$$

Energy conservation

$$\frac{3}{2}\frac{Dp}{Dt} + \frac{5}{2} p \vec{\nabla} \vec{u} = \boxed{\vec{J}\vec{E}^{*}}$$

We need Ohm's law

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Generalized Ohm's law: all terms

Ohmic term Hall term Ambipolar term

$$\vec{E}^* = \left[\vec{E} + \vec{u} \times \vec{B}\right] = \boxed{\eta \vec{J}} + \boxed{\eta_H \left[\vec{J} \times \vec{b}\right]} - \boxed{\eta_A \left[(\vec{J} \times \vec{b}) \times \vec{b}\right]}$$

$$+ \boxed{\frac{\varepsilon \vec{G} - \vec{\nabla}\hat{\mathbf{p}}_\mathbf{e}}{e n_e}} + \frac{\xi_n}{\alpha_n}\left[\vec{G} \times \vec{B}\right] + \frac{\xi_n^2 \rho_e}{\alpha_n} \vec{g} \times \vec{B} + \frac{m_e}{e}(1 + \xi_n \epsilon)\vec{g}$$

Biermann battery term …

Generalized Ohm's law: all terms

$$\vec{E}^* = \left[\vec{E} + \vec{u} \times \vec{B}\right] = \eta \vec{J} + \eta_H \left[\vec{J} \times \vec{b}\right] - \eta_A \left[(\vec{J} \times \vec{b}) \times \vec{b}\right]$$
$$+ \frac{\varepsilon \vec{G} - \vec{\nabla} \hat{\mathbf{p}}_{\mathbf{e}}}{e n_e} + \frac{\xi_n}{\alpha_n} \left[\vec{G} \times \vec{B}\right] + \frac{\xi_n^2 \rho_e}{\alpha_n} \vec{g} \times \vec{B} + \frac{m_e}{e} (1 + \xi_n \epsilon) \vec{g}$$

Definitions:

$$\vec{G} = \xi_n \vec{\nabla} \left(\hat{\mathbf{p}}_{\mathbf{i}} + \hat{\mathbf{p}}_{\mathbf{e}}\right) - \xi_i \vec{\nabla} \hat{\mathbf{p}}_{\mathbf{n}}$$ **- partial pressures term**

neutral fraction

ion fraction

$\vec{b}$ unity vector in the direction of mag. field;

$\alpha_n = \rho_i \nu_{in} + \rho_e \nu_{en}$ collisions

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Generalized Ohm's law: all terms

$$\vec{E}^* = \left[\vec{E} + \vec{u} \times \vec{B}\right] = \eta \vec{J} + \eta_H \left[\vec{J} \times \vec{b}\right] - \eta_A \left[(\vec{J} \times \vec{b}) \times \vec{b}\right]$$
$$+ \frac{\varepsilon \vec{G} - \vec{\nabla} \hat{\mathbf{p}}_\mathbf{e}}{e n_e} + \frac{\xi_n}{\alpha_n}\left[\vec{G} \times \vec{B}\right] + \frac{\xi_n^2 \rho_e}{\alpha_n} \vec{g} \times \vec{B} + \frac{m_e}{e}(1 + \xi_n \epsilon)\vec{g}$$

Assumptions:

Neglect electron inertia and momentum terms;

Quasi-neutrality;

Same temperatures of all species;

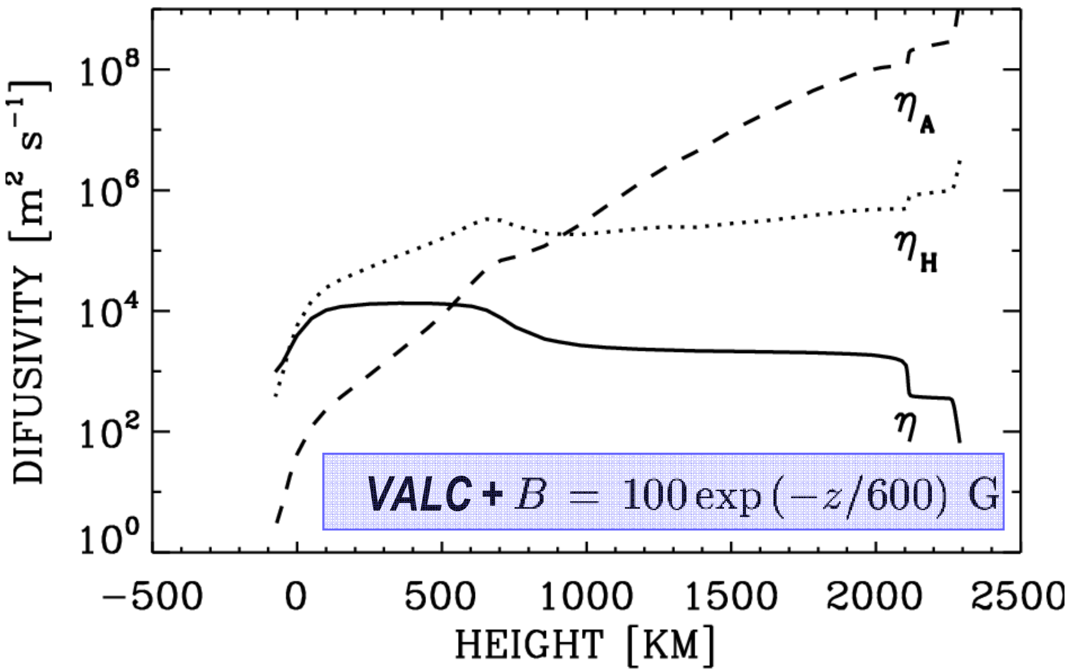
Time variations of diffusion velocity $\vec{w}_\alpha$ are neglected;

Pressure tensor has to be assumed scalar.

Generalized Ohm's law

$$\left[\vec{E} + \vec{u} \times \vec{B}\right] = \boxed{\eta \vec{J}} + \boxed{\eta_H \left[\vec{J} \times \vec{b}\right]} - \boxed{\eta_A \left[(\vec{J} \times \vec{b}) \times \vec{b}\right]}$$

Ohmic term | Hall term | Ambipolar term



$$\eta = \frac{m_e \nu_e}{e n_e^2}$$

$$\eta_H = \frac{|B|}{e n_e}$$

$$\eta_A = \frac{\xi_n^2 |B|^2}{\alpha_n}$$

Depends on neutron fraction, collision frequency and B.

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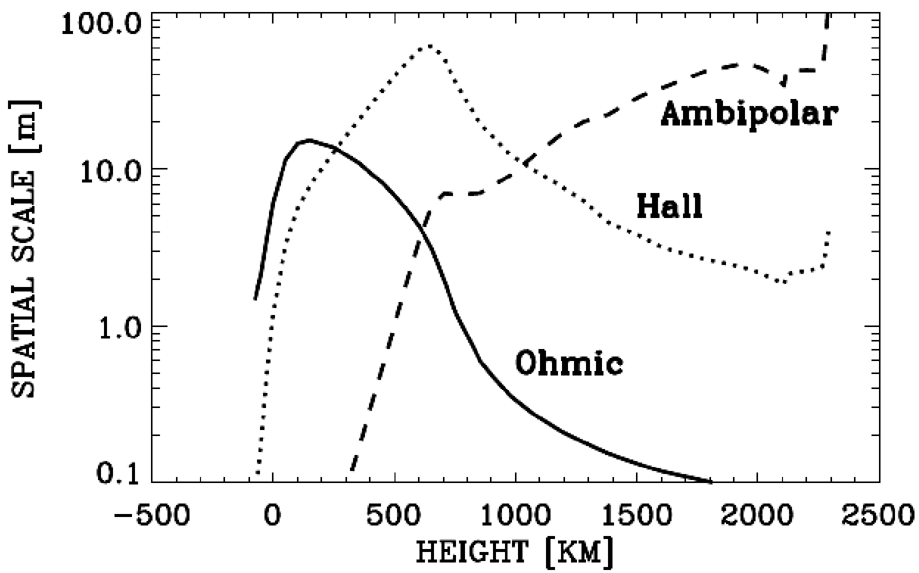
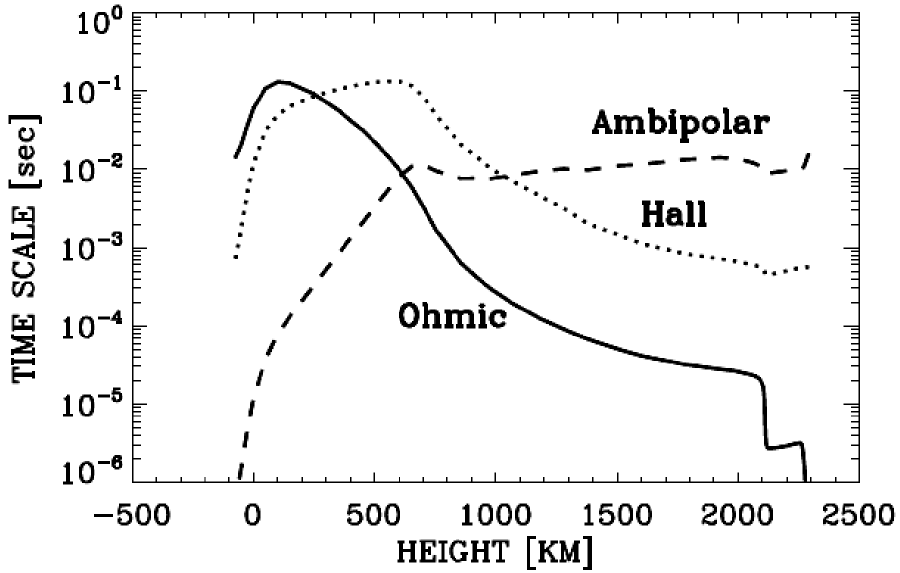
Generalized Ohm's law

spatial and temporal scales





VALC **+** $B = 100\,\exp\left(-z/600\right)\ \mathrm{G}$

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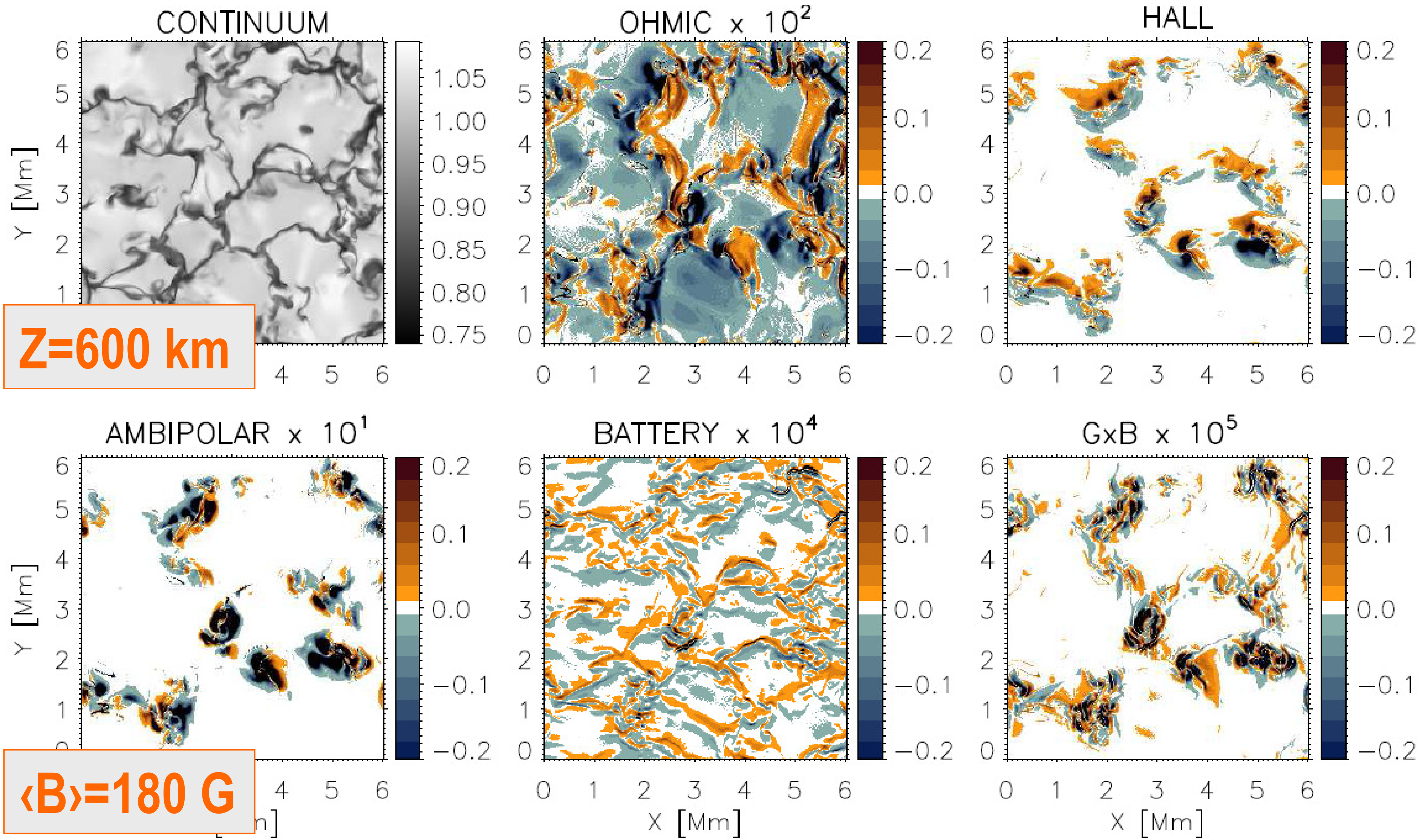
Generalized Ohm's law: all terms

Photospheric magneto-convection snapshot from Vögler et al (2005)

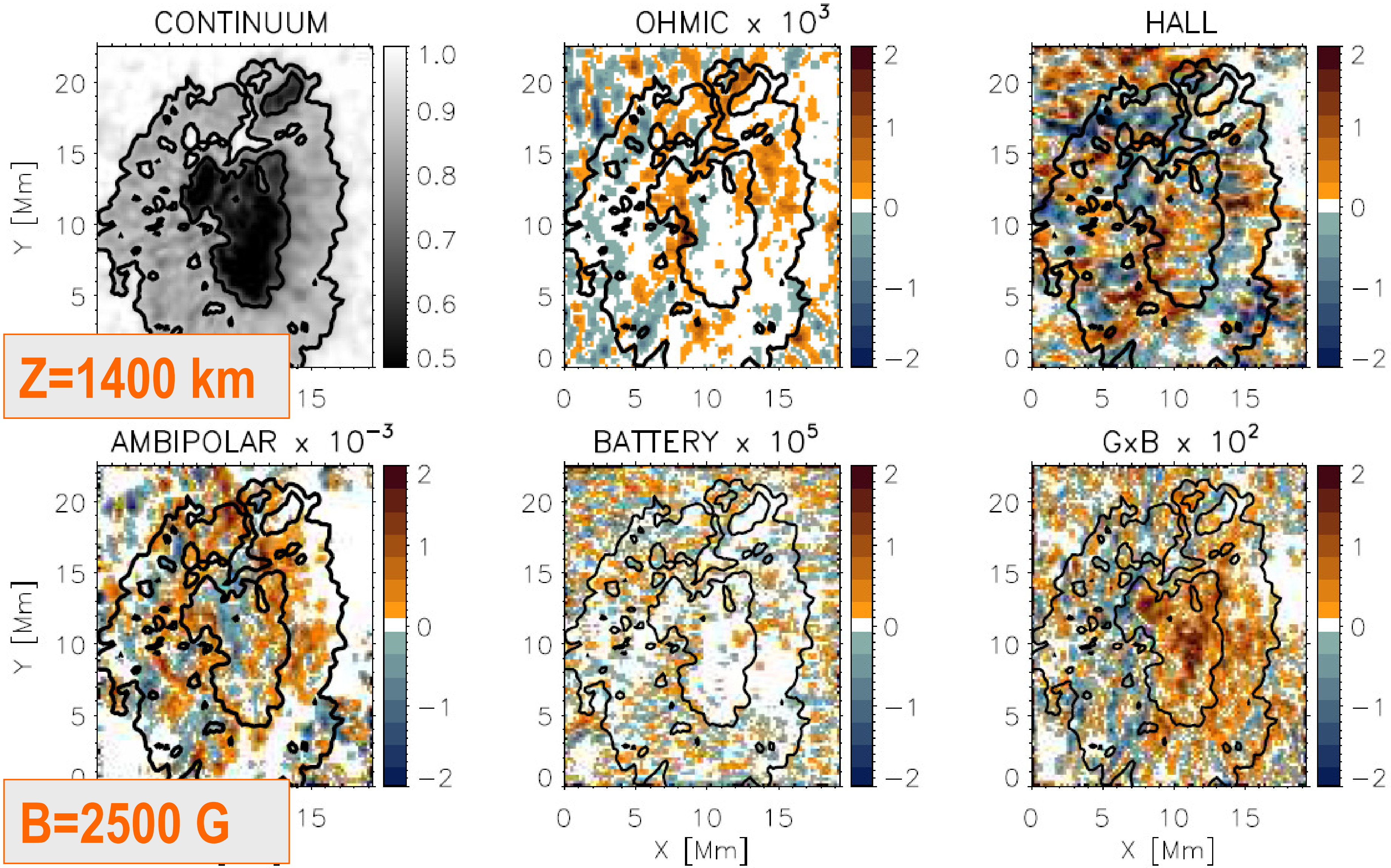


Z=600 km

⟨B⟩=180 G

Generalized Ohm's law: all terms

Inversion of chromospheric sunspot observations by Socas-Navarro (2005)



Two-fluid equations: definitions

see Bittencourt (1986)

$$\vec{u}_c = \frac{\rho_i \vec{u}_i + \rho_e \vec{u}_e}{\rho_i + \rho_e}$$

Center of mass velocity of charges

$$\hat{\mathbf{p}}_{ie} = \hat{\mathbf{p}}_i + \hat{\mathbf{p}}_e \sum_{\alpha=i,e} \rho_\alpha \vec{w}_\alpha \otimes \vec{w}_\alpha$$

Ion-electron kinetic pressure tensor

$$\begin{aligned} \vec{w}_i &= \vec{u}_i - \vec{u}_c \\ \vec{w}_e &= \vec{u}_e - \vec{u}_c \end{aligned}$$

Diffusion velocities of charges

Two-fluid equations

see Bittencourt (1986)

c = (e + i) = (electrons + ions)

Mass conservation for charged species

$$\frac{\partial \rho_c}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c) = S_{ie}$$

Momentum conservation for charged species

$$\frac{\partial \rho_c \vec{u}_c}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c \otimes \vec{u}_c) = [\vec{J} \times \vec{B}] + \rho_c \vec{g} - \nabla \hat{\mathbf{p}}_{\mathbf{ie}} + \vec{R}_{ie}$$

Energy conservation for charged species

$$\frac{3}{2}\frac{\partial p_{ie}}{\partial t} + \frac{3}{2}\vec{\nabla}(\vec{u}_c p_{ie}) + (\hat{\mathbf{p}}_{\mathbf{ie}}\vec{\nabla})\vec{u}_c + \vec{\nabla}\vec{q}_{ie} =$$

$$= \vec{J}[\vec{E} + \vec{u}_c \times \vec{B}] + \frac{1}{2}u_c^2 S_{ie} + \vec{u}_c \vec{R}_{ie} + M_{ie}$$

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Two-fluid equations

see Bittencourt (1986)

c = (e + i) = (electrons + ions)

Mass conservation for charged species

$$\frac{\partial \rho_c}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c) = \cancel{S_{ie}}$$

Momentum conservation for charged species

$$\frac{\partial \rho_c \vec{u}_c}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c \otimes \vec{u}_c) = [\vec{J} \times \vec{B}] + \rho_c \vec{g} - \nabla \hat{\mathbf{p}}_{\mathbf{ie}} + \vec{R}_{ie}$$

$$\boxed{\hat{\mathbf{p}}_{ie} = p_{ie}}$$

Energy conservation for charged species

$$\frac{3}{2}\frac{\partial p_{ie}}{\partial t} + \frac{3}{2}\vec{\nabla}(\vec{u}_c p_{ie}) + (\hat{\mathbf{p}}_{\mathbf{ie}}\vec{\nabla})\vec{u}_c + \cancel{\vec{\nabla}\vec{q}_{ie}} =$$

$$= \vec{J}[\vec{E} + \vec{u}_c \times \vec{B}] + \frac{1}{2}\cancel{u_c^2 S_{ie}} + \vec{u}_c \vec{R}_{ie} + \cancel{M_{ie}}$$

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Two-fluid equations

$$\vec{R}_{ie} = -\alpha_n(\vec{u}_c - \vec{u}_n) - \frac{\rho_n}{\rho_c}\frac{\vec{J}}{e}(m_e\nu_{ni} - m_i\nu_{ne})$$

Mass conservation for charged species

$$\frac{\partial \rho_c}{\partial t} + \vec{\nabla}\left(\rho_c \vec{u}_c\right) \quad = \quad 0$$

Momentum conservation for charged species

$$\frac{\partial \rho_c \vec{u}_c}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c \otimes \vec{u}_c) \quad = \quad [\vec{J} \times \vec{B}] + \rho_c \vec{g} - \nabla p_{ie} \boxed{+ \vec{R}_{ie}}$$

Energy conservation for charged species

$$\frac{3}{2}\frac{Dp_{ie}}{Dt} + \frac{5}{2}(p_{ie}\vec{\nabla})\vec{u}_c \quad = \quad \vec{J}[\vec{E} + \vec{u}_c \times \vec{B}] \boxed{+ \vec{u}_c \vec{R}_{ie}}$$

Transport theory in partially ionized plasmas

How can we compute

Collisional frequencies;

Elements of the pressure tensor;

Heat flux vector;

Ionization-recombination terms;

Radiative transfer terms ?

Numerical solution

How can we solve the 2-fluid equations numerically?

Special treatment for collisional terms?

$$\boxed{\vec{R}_{ie} = -\alpha_n(\vec{u}_c - \vec{u}_n) - \frac{\rho_n}{\rho_c}\frac{\vec{J}}{e}(m_e\nu_{ni} - m_i\nu_{ne})}$$

How much new physics do we gain in 2-fluid compared to quasi-MHD?

Where can it be important?

Wave propagation

Force balance

Magnetic reconnection

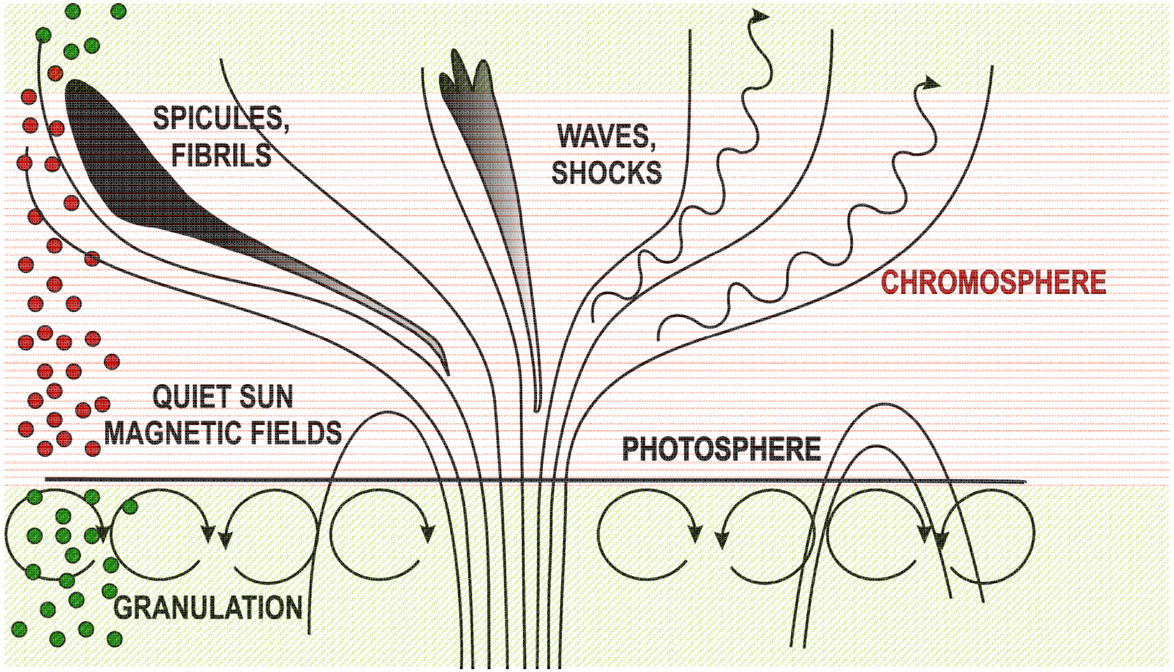
Flux emergence

Chromospheric heating

Interstellar medium

Shock fronts

Turbulence…etc.



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