Radiative transfer in (solar) multi-fluid and MHD simulations

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Radiation

Radiation:

- important source of heating and cooling,
- main source of information about astrophysical plasmas.

MHD and MF numerical simulations: omnipresent numerical laboratories.

Radiative transfer is essential for MHD and MF simulations.

Realistic numerical simulations in 3D: standard tool, still challenging for both physics, mathematics and programming.

Energy conservation equation

MHD:

$$\frac{\partial e}{\partial t} + \nabla \cdot \left[\mathrm{v} \left(e + p + \frac{|\mathrm{B}|^2}{8\pi} \right) - \frac{1}{4\pi} \mathrm{B}(\mathrm{v} \cdot \mathrm{B}) \right] =$$

$$= \frac{1}{4\pi} \nabla \cdot (\mathrm{B} \times \eta \nabla \times \mathrm{B}) + \nabla \cdot (\mathrm{v} \cdot \tau) + \nabla \cdot (K \nabla T) + \rho(\mathrm{g} \cdot \mathrm{v}) + Q_{\mathrm{rad}}$$

Multi-fluid:

$$\frac{3}{2} \frac{Dp}{Dt} + \frac{3}{2} \mathsf{p} \nabla \cdot \mathrm{u} + \mathrm{q} = \sum_{\alpha} (\omega_{\alpha} \cdot \nabla) \mathsf{p}_{\alpha} + \sum_{\alpha} Q_{\alpha} + Q_{\mathrm{rad}}$$

where **radiative energy exchange** is:

$$Q_{\mathrm{rad}} = - \int_{\nu} (\nabla \cdot \vec{F}_{\nu}) d\nu$$

$$Q_{\mathrm{rad}} = 4\pi\kappa\rho \int_{\nu} \kappa_{\nu} (J_{\nu} - B_{\nu}) d\nu$$

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Intensity, mean intensity, flux

Mean intensity:

$$J_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu(\mu)d\omega$$

Flux:

$$\vec{F}_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu(\vec{\mu})\vec{\mu}d\omega$$

Specific intensity:

$$de_\nu = I(\vec{r},\vec{\mu},t,\nu)dA\cos\theta\ d\omega d\nu dt$$

$I(\vec{r},\vec{\mu},t,\nu)$: 3 spatial coordinates, 2 angles, frequency, time.

Radiative transfer equation

RTE:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \vec{\mu}\cdot\nabla I_\nu = j_\nu^{\rm tot} - \kappa_\nu^{\rm tot} I_\nu$$

$\partial I_\nu/\partial t$ can be neglected for non-relativistic fluids:

$$\vec{\mu}\cdot\nabla I_\nu = j_\nu^{\rm tot} - \kappa_\nu^{\rm tot} I_\nu$$

In plan-parallel 1D case:

$$-\frac{dI_\nu}{\kappa_\nu \rho dz} = \frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

where S_ν is source function:

$$S_\nu = (1-\varepsilon_\nu)J_\nu + \varepsilon_\nu B_\nu$$

where ε_ν is photon distraction probability.

RT schemes and requirements

Requirements for RT scheme (e.g. see Davis et al, 2012):

- periodic boundaries,
- T and ρ discontinuities
- explicit form of J_ν (for NLTE)
- efficient for simple problems where RT does not dominate
- suitable for domain decomposition

The most common RT schemes

- Flux limited diffusion
- Ray tracing: short and long characteristics

Key issue: discretization frequency, spatial, angular.

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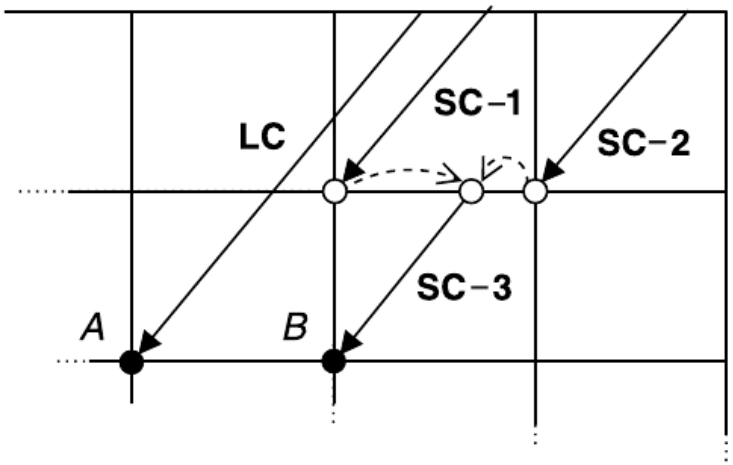
Codes

code	grid	(N)LTE	RT solver	rays	bins
MURaM*	uniform	LTE	Short	12	4
STAGGER	uniform	LTE	Long ch.	9	12
Co5Bold	uniform	LTE	Long	17	12
BIFROST	non-uni.	NLTE	Short		
Athena	uniform	NLTE	Short		
Flash	AMR		FLD		

STAGGER (Nordlund & Galsgaard 1995; Carlsson et al. 2004; Stein & Nordlund 2006), MURaM (Vögler, 2004; Rempel et al, 2009), Co5Bold (Freytag et al, 2002; Wedemeyer et al, 2004), BIFROST (Gudiksen et al, 2011; Hayek et al, 2010), ATHENA (Stone et al, 2008; Davis et al, 2012); Flash (Linde, 2002)

* The MANCHA code (Felipe et al, 2011) $\approx$ MURaM.

Ray tracing



- Long characteristics (see Feautrier, 1964; Heinemann et al, 2006): more computationally expensive, more difficult to use with domain decomposition
- Short characteristics (Mihalas et al, 1978; Olson and Kunasz, 1987): more numerical diffusion

$$J = \sum_{k=1}^{N_{\rm ang}} w_k I_k \qquad\qquad F_i = \sum_{k=1}^{N_{\rm ang}} w_k \mu_{ik} I_k$$

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Short characteristics

Formal solution of $\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$:

$$I_\nu(\tau_\nu) = I_\nu(\tau_\nu^0)\mathrm{e}^{-(\tau_\nu - \tau_\nu^0)} + \int_{\tau_\nu^0}^{\tau_\nu} S_\nu \mathrm{e}^{-(\tau_\nu - t_\nu)} dt_\nu$$

- LTE: $S_\nu := B_\nu(T_{\mathrm{MHD}})$
  - Example: MURaM (Vögler, 2004)
- NLTE: iteration procedure for S_ν and J_ν
  - Examples: van Noort et al (2002), Hayek et al (2010), Davis et al (2012)
  - Accelerated Lambda Iteration (e.g. Gauss-Seidel by Trujillo Bueno & Fabiani Bendicho, 1995)

MURaM

The MURaM code (Vögler, 2004)

- fully compressible MHD;
- time-dependent, uniform 3D Cartesian grid;
- non-local, LTE, non-gray radiative transfer solved by short characteristics;
- realistic equation of state including partial ionization;
- MPI parallelized.

The code has been used to simulate quiet sun, plage, umbra, active regions and sunspot (and to study phenomena as local dynamo, flux emergence, dynamics of the solar photosphere).

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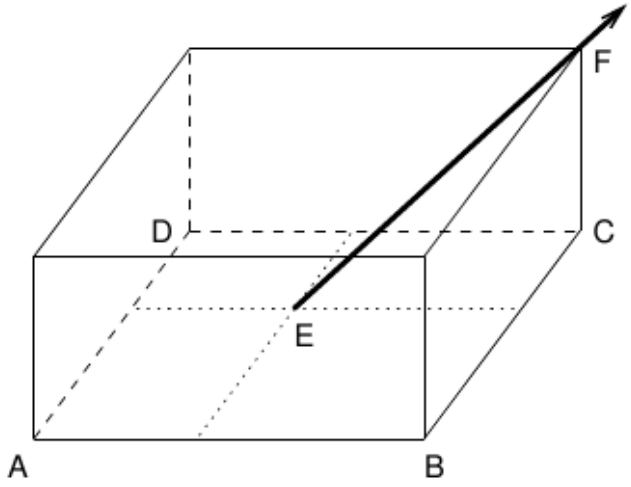
Short characteristics in MURaM



Formal solution for interval EF:

$$I_F = I_E \, \mathrm{e}^{\Delta\tau_{EF}} + \int_{\tau_F}^{\tau_E} B(\tau) \, \mathrm{e}^{\tau_F - \tau} dt \qquad \Delta\tau_{EF} = \int_F^E \kappa(s)\rho(s) ds$$

- I_E from bilinear interpolation
- $I_{A,B,C,D}$ a priori unknown, extrapolated from previous time steps
- ρ, κ, B linear at EF
- 3 rays per octant

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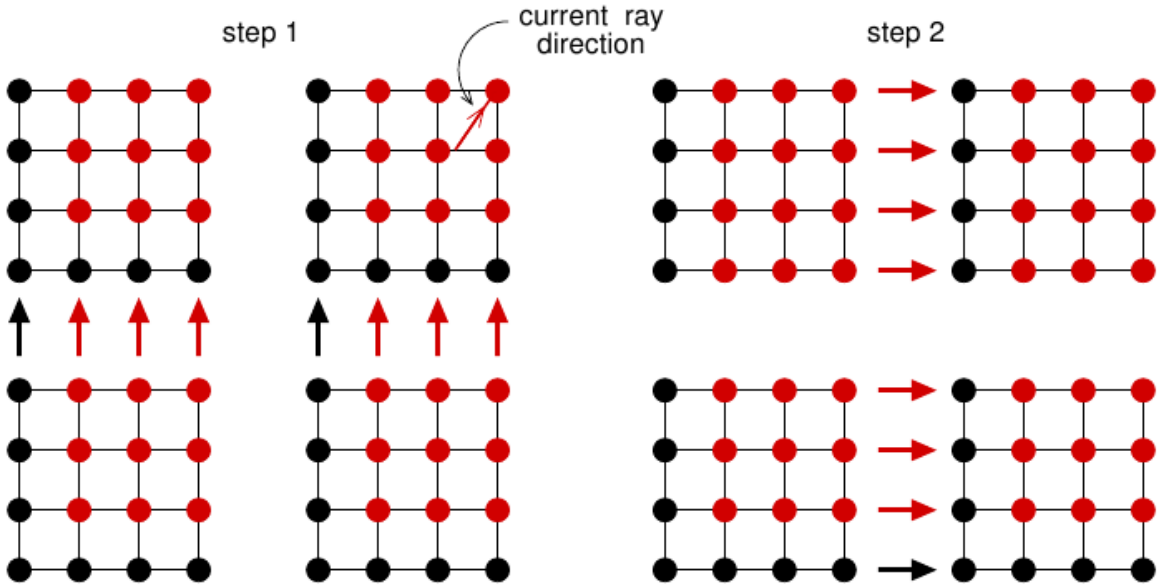
Short characteristics in MURaM, cont.



At global boundaries:

- Top: $I_{\nu\mu}^{\mathrm{top}} = 0$
- Top (opaque ν): $I_{\nu\mu}^{\mathrm{top}} = B_\nu(T_{\mathrm{top}})(1 - \mathrm{e}^{\tau_{\mathrm{top}}/\mu})$
- Bottom: $I_{\nu\mu}^{\mathrm{bottom}} = B_\nu$

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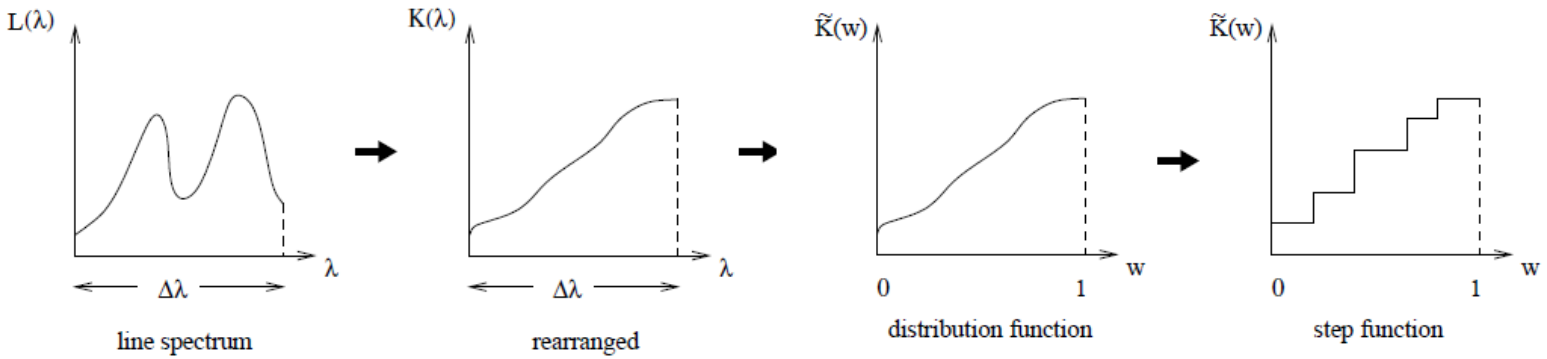
Frequency discretization

- Frequency discretization: $10^6 - 10^7$ points to cover the wavelength range [50 nm, 10 m].(Carlsson, 2004)
- Methods to reduced number of ν points: grey approximation, **opacity binning**, opacity distribution function, opacity sampling.
- $$\frac{dI_i}{dz} = \int_{\Omega_i} \kappa_\nu \rho (B_\nu - I_\nu) d\nu \approx \overline{\kappa_i} \rho (B_i - I_i)$$
- How many bins is sufficient?
- Co5bold, Stagger, MANCHA



Flux limited diffusion

Goal: to compute F_ν and J_ν without solving RTE for I_ν.
RTE (assuming isotropic scattering):

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \nabla \cdot (\vec{\mu} I_\nu) = j_\nu - \kappa_\nu I_\nu$$

First moment:

$$\frac{\partial J_\nu}{\partial t} + \nabla \cdot \vec{F_\nu} = 4\pi j_\nu - \kappa_\nu c J_\nu$$

Second moment:

$$\frac{1}{c}\frac{\partial \vec{F_\nu}}{\partial t} + c\nabla \cdot \mathsf{P}_\nu = -\kappa_\nu \vec{F_\nu}$$

Eddington's approximation ($\kappa_\nu L \gg 1$):

$$\mathsf{P}_\nu = \frac{1}{3} J_\nu \mathsf{I}$$

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Flux limited diffusion, cont.

First moment:

$$\frac{\partial J_\nu}{\partial t} + \nabla \cdot \vec{F_\nu} = 4\pi j_\nu - \kappa_\nu c J_\nu$$

Second moment + Eddington's approximation:

$$\frac{1}{c}\frac{\partial \vec{F_\nu}}{\partial t} + \frac{c}{3}\nabla J_\nu = -\kappa_\nu \vec{F_\nu}$$

$\partial/\partial t$ of the 1st moment + ∇ of the 2nd + EA (and omitting j_ν and κ_ν terms):

$$\frac{\partial^2 J_\nu}{\partial t^2} - \frac{c^2}{3}\nabla^2 J_\nu = 0$$

Wave speed $c/\sqrt{3}$ - wrong!

Flux limited diffusion, cont.

Instead, we omit $\partial \vec{F}_\nu / \partial t$:

$$\frac{c}{3} \nabla J_\nu = -\kappa_\nu \vec{F}_\nu$$

and substitute to the 1st moment eq.

$$\frac{\partial J_\nu}{\partial t} - \nabla \cdot \left(\frac{c}{3\kappa_\nu} \nabla J_\nu \right) = 4\pi j_\nu - \kappa_\nu c J_\nu$$

To avoid propagation speeds greater than c (and to limit flux that became arbitrarily large for large ∇J_ν) a correction factor (*flux limiter* D) is used:

$$\vec{F}_\nu = -\frac{c\mathrm{D}}{\kappa_\nu \rho} \nabla J_\nu$$

Flux limited diffusion, cont.

So defined flux limiter is arbitrary, ad hoc, function of

$$R = \frac{|\nabla J_\nu|}{\kappa_\nu \rho J_\nu}.$$

Levermore & Pomraning (1981) added ε_ν to denominator or R and defined D as:

$$\mathsf{D} = \frac{1}{R}\left(\coth R - \frac{1}{R}\right)$$

Nordlund's criticism of FLD

Nordlund (2011) tested the ray tracing with long characteristics (RTLC) versus the flux limited diffusion (example: fragmented stellar disc at low T):

- FLD is an approximation that does not converge to the exact solution, while RTLC does it as the number of rays increases.
- FLD reduces number of variables from 6 to 4, but computational cost for eliptic equation in J is significant.
- RTLC easier to implement with "near-perfect" parallelization properties.
- Result of the test: RMS error of FLD largest around $\tau = 1$ (reaches 0.4).

How universal are these conclusions?

Some conclusions and some (unanswered) questions

Ray tracing (short characteristics) appear as the optimal choice for a MF/MHD code modellling solar photo/chromosphere as long as the grid is **regular**.

Multi-fluid approach is likely to require multiresolution.

- Different grids for MHD/MF and RT?
- One AMR for many frequencies?
- Can FLD account for continuum scattering?
- How to adapt SC for adaptive mesh grid?
- Is SC still superior than FLD in that case?
- Would it be possible to combine best of both methods?
- What are the alternatives?

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Some “unconventional” aproaches

- Dedner and Vollmoeller (2002):
  - introduced short characteristics in a finite element framework;
  - multiresolution, unstructured, triangular grid;
  - SC applicabble only in the first order and too dissipative;
  - not clear how to proceed to 3D from there.
  - see also Bruls et al (2006):short characteristics with unstructured triangular grid.

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Some “unconventional” aproaches

- Hübner and Turek (2007):
  - a “very mathematical” paper;
  - short characteristics;
  - extension of ALI, generalized mean intensity;
  - highly unstructured meshes.

- Juvela and Padoan (2005):
  - MHD simulation interstellar clouds with AMR;
  - separate grid for RT;
  - NLTE: ALI + cobined long short characteristics;
  - optimized memory use, not in parallel (?).

Some “unconventional” aproaches

- Meier (1999): Finite elements:
  - adaptive mesh can extend into the time domain;
  - equations written in a compact form on a simple grid;
  - computationally expensive;
  - for fluid - finite volumes.

- Richling et al (2001):
  - Radiative transfer with Meier’s finite elements.
  - Comparison to Monte Carlo.