Two-fluid MHD approach for partially ionized solar plasmas

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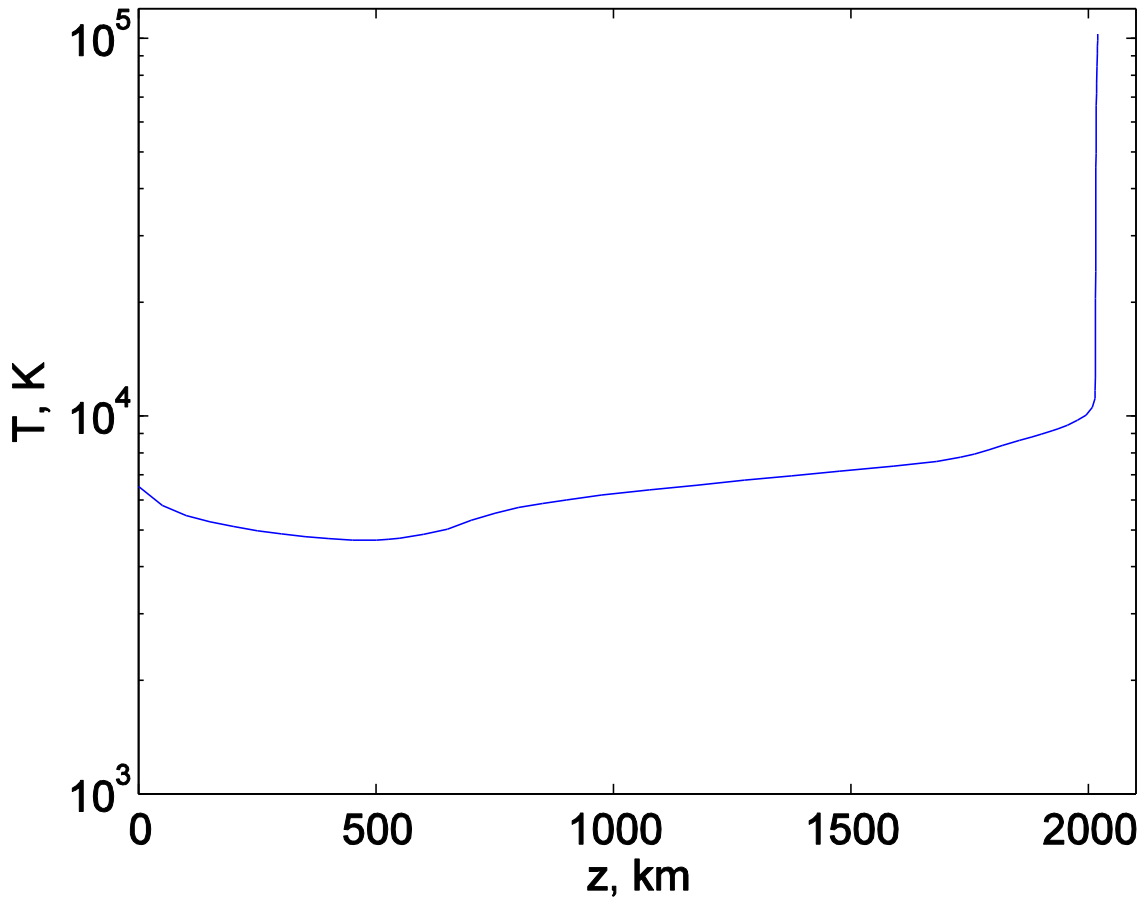
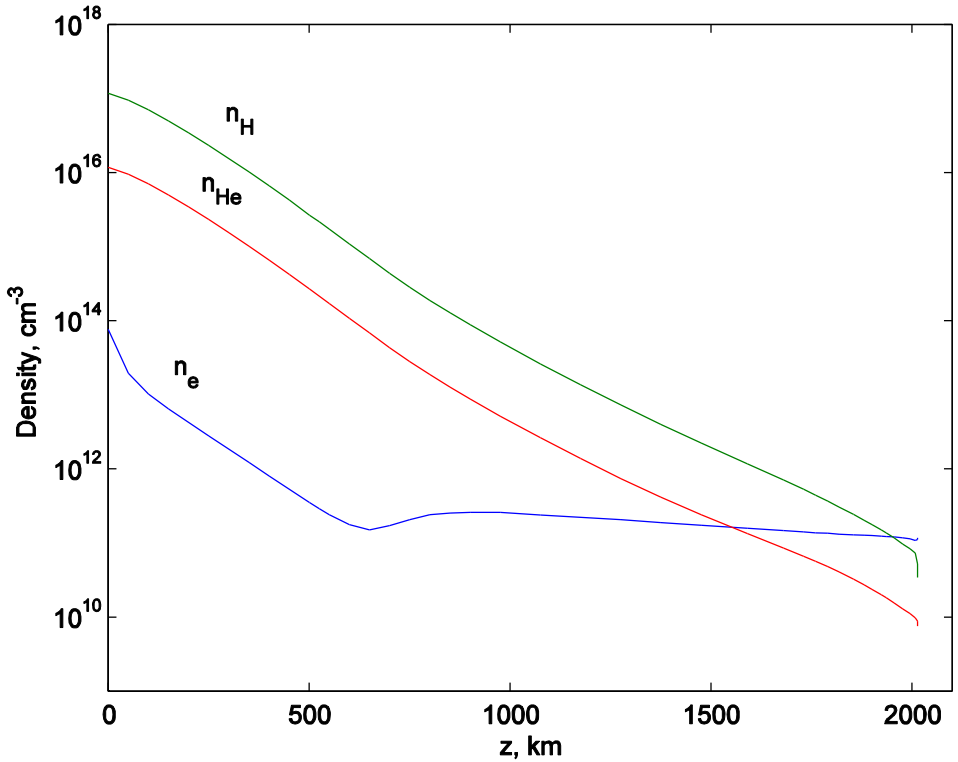
Solar atmospheric model



FAL93-3 model (Fontenla et al. 1993)

Solar atmospheric model



FAL93-3 model (Fontenla et al. 1993)

n_e – electron number density

n_H – neutral hydrogen number density

n_{He} – neutral helium number density

Plasma is only weakly ionized in the photosphere, but becomes almost fully ionized in the transition region and corona.

Effects of neutral atoms in the solar atmosphere

Neutral atoms may change the plasma dynamics through collisions with charged particles:

Damping of MHD waves (Braginkii 1965, De Pontieu et al. 2001, Khodachenko et al. 2004, Leak et al. 2006, Forteza et al. 2007, Soler et al. 2009, 2010, Carbonell et al. 2010, Singh and Krishan 2010, Zaqarashvili et al. 2011a,b) ;

Formation of spicules (Haerendel 1992, De Pontieu & Haerendel 1998, James & Erdélyi 2002, James et al. 2004);

Influence on energy flux of Alfvén waves in the photosphere (Vranjes et al. 2008);

Generation of electric currents and plasma heating (Sen and White 1972, Khodachenko & Zaitsev 2002, Fontenla et al. 2008, Gogoberidze et al. 2009, Krasnoselskikh et al. 2010, Khomenko and Collados, 2012a,b);

Emerging magnetic flux tubes (Leake & Arber 2005, Arber et al. 2007);

Influence on resonant absorption (Soler et al. 2009).

Influence on Kelvin-Helmholtz instability (Soler et al. 2012).

Three-Fluid equations

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{V}_e) = 0,$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{V}_i) = 0,$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \vec{V}_n) = 0,$$

$$m_e n_e \left(\frac{\partial \vec{V}_e}{\partial t} + (\vec{V}_e \cdot \nabla)\vec{V}_e \right) = -\nabla p_e - \nabla \cdot \pi_e - e n_e \left(\vec{E} + \frac{1}{c} \vec{V}_e \times \vec{B} \right) + \vec{R}_e,$$

$$m_i n_i \left(\frac{\partial \vec{V}_i}{\partial t} + (\vec{V}_i \cdot \nabla)\vec{V}_i \right) = -\nabla p_i - \nabla \cdot \pi_i - e n_i \left(\vec{E} + \frac{1}{c} \vec{V}_i \times \vec{B} \right) + \vec{R}_i,$$

$$m_n n_n \left(\frac{\partial \vec{V}_n}{\partial t} + (\vec{V}_n \cdot \nabla)\vec{V}_n \right) = -\nabla p_n - \nabla \cdot \pi_n + \vec{R}_n,$$

$$\frac{3}{2} n_e k \left(\frac{\partial T_e}{\partial t} + (\vec{V}_e \cdot \nabla) T_e \right) + p_e \nabla \cdot \vec{V}_e + \pi_e : \nabla \vec{V}_e = -\nabla \cdot \vec{q}_e + Q_e,$$

$$\frac{3}{2} n_i k \left(\frac{\partial T_i}{\partial t} + (\vec{V}_i \cdot \nabla) T_i \right) + p_i \nabla \cdot \vec{V}_i + \pi_i : \nabla \vec{V}_i = -\nabla \cdot \vec{q}_i + Q_i,$$

$$\frac{3}{2} n_n k \left(\frac{\partial T_n}{\partial t} + (\vec{V}_n \cdot \nabla) T_n \right) + p_n \nabla \cdot \vec{V}_n + \pi_n : \nabla \vec{V}_n = -\nabla \cdot \vec{q}_n + Q_n,$$

$$p_e = n_e k T_e,\ p_i = n_i k T_i,\ p_n = n_n k T_n,$$

We consider partially ionized plasma, which consists of electrons (e), protons (i) and neutral hydrogen (n).

(Braginski 1965)

$\vec{R}_a$ is the change of impulse, $\vec{q}_a$ is the heat flux density, Q_a is the heat production.

Maxwell equations

The description of the system is completed by Maxwell equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t},$$

$$\nabla \times \vec{B} = -\frac{4\pi}{c}\vec{j},$$

where

$$\vec{j} = -en_e\left(\vec{V}_e - \vec{V}_i\right)$$

is the current density and

$$\nabla \cdot \vec{B} = 0.$$

Plasma is supposed to be quasi neutral

$$n_e = n_i.$$

Impulse change and heat production

Impulse change and heat production can be expressed as (Braginskii 1965)

$$\vec{R}_e = -\alpha_{ei}\left(\vec{V}_e - \vec{V}_i\right) - \alpha_{en}\left(\vec{V}_e - \vec{V}_n\right),$$
$$\vec{R}_i = -\alpha_{ie}\left(\vec{V}_i - \vec{V}_e\right) - \alpha_{in}\left(\vec{V}_i - \vec{V}_n\right),$$
$$\vec{R}_i = -\alpha_{ne}\left(\vec{V}_n - \vec{V}_e\right) - \alpha_{ni}\left(\vec{V}_n - \vec{V}_i\right),$$
$$Q_e = \alpha_{ei}\left(\vec{V}_e - \vec{V}_i\right)\vec{V}_e + \alpha_{en}\left(\vec{V}_e - \vec{V}_n\right)\vec{V}_e,$$
$$Q_i = \alpha_{ie}\left(\vec{V}_i - \vec{V}_e\right)\vec{V}_i + \alpha_{in}\left(\vec{V}_i - \vec{V}_n\right)\vec{V}_i,$$
$$Q_n = \alpha_{ne}\left(\vec{V}_n - \vec{V}_e\right)\vec{V}_n + \alpha_{ni}\left(\vec{V}_n - \vec{V}_i\right)\vec{V}_n,$$

$\alpha_{ab} = \alpha_{ba}$ are coefficients of friction between different sort of particles.

For time scales longer than ion-electron collision time, the ion-electron gas can be considered as a single fluid.

Then, any additional sort of neutral atoms can be treated as a separate fluid.

Friction coefficients

The coefficient of friction between ions and electrons can be expressed as (Braginskii 1965)

$$\alpha_{ie} = \frac{4\sqrt{2\pi}\lambda e^4 n_i n_e m_{ie}}{3\sqrt{m_{ie}}\left(kT_e\right)^{3/2}},$$

where λ is the Coulomb logarithm .

The coefficient of friction between ions and neutral hydrogen atoms is (Braginskii 1965)

$$\alpha_{in} = n_i n_n m_{in} \sigma_{in} \sqrt{\frac{8kT}{\pi m_{in}}},$$

where σ_{in} is ion-hydrogen collision cross-section and m_{in} is the reduced mass.

For elastic collision, the ion-hydrogen collision cross-section is (Braginskii 1965)

$$\sigma_{in} = 2\pi\left(r_i + r_n\right)^2,$$

which approximately equals atomic cross section.

Collision frequencies

Ion-electron collision frequency is expressed by

$$\nu_{ie} = \frac{\alpha_{ie}}{m_e n_e} = \frac{4\sqrt{2\pi}\lambda e^4 n_i}{3\sqrt{m_e}\left(kT_e\right)^{3/2}}.$$

Ion-neutral collision frequency is often expressed as

$$\nu_{in} = \frac{\alpha_{in}}{m_i n_i},$$

while neutral-ion collision frequency is often expressed as

$$\nu_{ni} = \frac{\alpha_{in}}{m_n n_n}.$$

According to this formulation, ion-neutral collision frequency is different than neutral-ion collision frequency. But, the mean collision frequency between ions and neutral atoms should be a single value due to physical basis.

From simple equations of motion of ions and neutrals one can derive the equation for relative velocity between ions and neutrals

$$\frac{\partial\left(\vec{V}_i - \vec{V}_n\right)}{\partial t} = -\alpha_{in}\left(\frac{1}{m_i n_i} + \frac{1}{m_n n_n}\right)\left(\vec{V}_i - \vec{V}_n\right).$$

This equation gives a single value for the ion-neutral collision frequency (Zaqarashvili et al. 2011)

$$\nu_{in} = \nu_{ni} = \alpha_{in}\left(\frac{1}{m_i n_i} + \frac{1}{m_n n_n}\right) = 2(n_i + n_n)\sigma_{in}\sqrt{\frac{kT}{\pi m_i}}.$$

Collision frequencies in the solar atmosphere

Let us use FAL93-3 model (Fontenla et al. 1993) for temperature and number densities:

z=0:	z=900:	z=1900:
T=6520 K	T=6000 K	T=8900 K
$n_e = 7.67\ 10^{13}\ cm^{-3}$	$n_e = 2.60\ 10^{11}\ cm^{-3}$	$n_e = 1.27\ 10^{11}\ cm^{-3}$
$n_i = 5.99\ 10^{13}\ cm^{-3}$	$n_i = 2.43\ 10^{11}\ cm^{-3}$	$n_i = 1.20\ 10^{11}\ cm^{-3}$
$n_n = 1.18\ 10^{17} cm^{-3}$	$n_n = 8.95\ 10^{13} cm^{-3}$	$n_n = 1.79\ 10^{11} cm^{-3}$
$\nu_{ie} = 8\ 10^{8}$ Hz	10^{6} Hz	$3.7\ 10^{6}$ Hz
$\nu_{in} = 8.6\ 10^{6}$ Hz	$6.2\ 10^{3}$ Hz	24 Hz

Ion-electron collision frequency seems to be always much higher than ion-neutral collision frequency in the whole atmosphere.

However, this statement may be overestimated in the lower photosphere, where significant number of heavy ions exists.

Two-Fluid MHD equations

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{V}_i) = 0,$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \vec{V}_n) = 0,$$

$$m_i n_i \left(\frac{\partial \vec{V}_i}{\partial t} + (\vec{V}_i \cdot \nabla)\vec{V}_i \right) = -\nabla p_{ie} - \frac{1}{c} \vec{j} \times \vec{B} + \frac{\alpha_{en}}{en_e} \vec{j} - (\alpha_{in} + \alpha_{en})(\vec{V}_i - \vec{V}_n),$$

$$m_n n_n \left(\frac{\partial \vec{V}_n}{\partial t} + (\vec{V}_n \cdot \nabla)\vec{V}_n \right) = -\nabla p_n - \frac{\alpha_{en}}{en_e} \vec{j} + (\alpha_{in} + \alpha_{en})(\vec{V}_i - \vec{V}_n),$$

(Zaqarashvili et al. 2011)

$$\frac{\partial p_{ie}}{\partial t} + (\vec{V}_i \cdot \nabla) p_{ie} + \gamma p_{ie} \nabla \cdot \vec{V}_i = (\gamma - 1) \frac{\alpha_{ei}}{e^2 n_e^2} j^2 + (\gamma - 1)\alpha_{in} (\vec{V}_i - \vec{V}_n) \cdot \vec{V}_i +$$

$$+ (\gamma - 1)\alpha_{en} (\vec{V}_e - \vec{V}_n) \cdot \vec{V}_e + \frac{(\vec{j} \cdot \nabla) p_e}{en_e} + \gamma p_e \nabla \cdot \frac{\vec{j}}{en_e} - (\gamma - 1) \nabla \cdot (\vec{q}_i + \vec{q}_e),$$

$$\frac{\partial p_n}{\partial t} + (\vec{V}_n \cdot \nabla) p_n + \gamma p_n \nabla \cdot \vec{V}_n = -(\gamma - 1)\alpha_{in} (\vec{V}_i - \vec{V}_n) \cdot \vec{V}_n - (\gamma - 1)\alpha_{en} (\vec{V}_e - \vec{V}_n) \cdot \vec{V}_n - (\gamma - 1) \nabla \cdot \vec{q}_n,$$

$p_{ie} = p_i + p_e$ is the pressure of electron-ion gas.

Induction equation in two-fluid approach

Ohm's law is obtained from the electron equation after neglecting the electron inertia

$$\vec{E}+\frac{1}{c}\vec{V}_i\times\vec{B}+\frac{1}{en_e}\nabla p_e=\frac{\alpha_{ei}+\alpha_{en}}{e^2n_e^2}\left(\vec{V}_i-\vec{V}_n\right)+\frac{1}{cen_e}\vec{j}\times\vec{B}.$$

Faraday's law and Ohm's law lead to the induction equation

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times\left(\vec{V}_i\times\vec{B}\right)+\nabla\times\left(\frac{c\nabla p_e}{en_e}\right)-\nabla\times\left(\eta\nabla\times\vec{B}\right)-\nabla\times\left(\frac{\vec{j}\times\vec{B}}{en_e}\right)+\nabla\times\left(\frac{c\alpha_{en}\left(\vec{V}_i-\vec{V}_n\right)}{en_e}\right),$$

where

$$\eta=\frac{c^2}{4\pi\sigma}=\frac{c^2\left(\alpha_{ei}+\alpha_{en}\right)}{4\pi e^2n_e^2}$$

is the coefficient of magnetic diffusion.

Single-Fluid equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0,$$

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}\right) = -\nabla p + \frac{1}{c}\vec{j} \times \vec{B} - \nabla \cdot \left(\xi_i \xi_n \rho \vec{w}\vec{w}\right),$$

$$\frac{\partial \vec{w}}{\partial t} + (\vec{w} \cdot \nabla)\vec{V} + (\vec{V} \cdot \nabla)\vec{w} + \xi_n (\vec{w} \cdot \nabla)\vec{w} - (\vec{w} \cdot \nabla)\xi_i \vec{w} = -\left(\frac{\nabla p_{ie}}{\rho \xi_i} - \frac{\nabla p_n}{\rho \xi_n}\right) + \frac{1}{c\rho\xi_i}\vec{j} \times \vec{B} + \frac{\alpha_{en}}{en_e \rho \xi_i \xi_n}\vec{j} - \frac{\alpha_{in} + \alpha_{en}}{\rho \xi_i \xi_n}\vec{w},$$

$$\frac{\partial p}{\partial t} + (\vec{V} \cdot \nabla)p + \gamma p \nabla \cdot \vec{V} - \xi_i (\vec{w} \cdot \nabla)p - \gamma p \nabla \cdot \left(\xi_i \vec{w}\right) + (\vec{w} \cdot \nabla)p_{ie} + \gamma p_{ie} \nabla \cdot \vec{w} = (\gamma - 1)\frac{\alpha_{ei} + \alpha_{en}}{e^2 n_e^2} j^2 + (\gamma - 1)(\alpha_{in} + \alpha_{en})w^2 -$$

$$-(\gamma - 1)\frac{2\alpha_{en}}{en_e}\vec{j}\vec{w} + \frac{\left(\vec{j} \cdot \nabla\right)p_e}{en_e} + \gamma p_e \nabla \cdot \frac{\vec{j}}{en_e} - (\gamma - 1)\nabla \cdot \left(\vec{q}_i + \vec{q}_e + \vec{q}_n\right),$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{V} \times \vec{B}\right) + \nabla \times \left(\frac{c\nabla p_e}{en_e}\right) - \nabla \times \left(\eta \nabla \times \vec{B}\right) - \nabla \times \left(\frac{\vec{j} \times \vec{B}}{en_e}\right) + \nabla \times \left(\frac{c\alpha_{en}\vec{w}}{en_e}\right) + \nabla \times \left(\xi_n \vec{w} \times \vec{B}\right),$$

where $\vec{V} = \dfrac{\rho_i \vec{V}_i + \rho_n \vec{V}_n}{\rho_i + \rho_n}$ is the total velocity, $\vec{w} = \vec{V}_i - \vec{V}_n$ is the relative velocity.

For time scales longer than ion-neutral collision time inertial terms can be neglected.

$$\vec{w} = -\frac{\vec{G}}{\alpha_{in} + \alpha_{en}} - \left(\frac{\nabla p_{ie}}{\rho \xi_i} - \frac{\nabla p_n}{\rho \xi_n}\right) + \frac{\xi_n}{c(\alpha_{in} + \alpha_{en})}\vec{j} \times \vec{B} + \frac{\alpha_{en}}{en_e(\alpha_{in} + \alpha_{en})}\vec{j},$$

Induction equation in single-fluid approach

Then we obtain

$$\vec{w} = -\frac{\vec{G}}{\alpha_{in}+\alpha_{en}} - \left(\frac{\nabla p_{ie}}{\rho\xi_i} - \frac{\nabla p_n}{\rho\xi_n}\right) + \frac{\xi_n}{c(\alpha_{in}+\alpha_{en})}\vec{j}\times\vec{B} + \frac{\alpha_{en}}{en_e(\alpha_{in}+\alpha_{en})}\vec{j},$$

and the induction equation becomes

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times\left(\vec{V}\times\vec{B}\right) + \frac{c}{e}\nabla\times\left(\frac{c\nabla p_e - \varepsilon\vec{G}}{n_e}\right) - \nabla\times\left(\eta_T\nabla\times\vec{B}\right) - \frac{c}{4\pi e}\nabla\times\left(\frac{1-2\varepsilon\xi_n}{n_e}\left(\nabla\times\vec{B}\right)\times\vec{B}\right) -$$
$$-\nabla\times\left(\frac{\xi_n}{\alpha_{in}+\alpha_{en}}\vec{G}\times\vec{B}\right) + \nabla\times\left(\frac{\xi_n^2}{4\pi(\alpha_{in}+\alpha_{en})}\left(\left(\nabla\times\vec{B}\right)\times\vec{B}\right)\times\vec{B}\right),$$

where

$$\eta_T = \frac{c^2}{4\pi e^2 n_e^2}\left(\alpha_{ei}+\alpha_{en}-\frac{\alpha_{en}^2}{\alpha_{in}+\alpha_{en}}\right), \qquad \vec{G} = \xi_n\nabla p_{ie} - \xi_i\nabla p_n \qquad \varepsilon = \frac{\alpha_{en}}{\alpha_{in}+\alpha_{en}}.$$

Next we consider MHD waves in the two-fluid approach.

Linear Alfvén waves in two-fluid approach

We consider linear Alfvén waves in homogeneous medium with uniform magnetic field

$$\frac{\partial u_{iy}}{\partial t} = \frac{B_z}{4\pi\rho_i}\frac{\partial b_y}{\partial z} - \frac{\alpha_{in}}{\rho_i}\left(u_{iy} - u_{ny}\right),$$

$$\frac{\partial u_{ny}}{\partial t} = \frac{\alpha_{in}}{\rho_n}\left(u_{iy} - u_{ny}\right),$$

$$\frac{\partial b_y}{\partial t} = B_z\frac{\partial u_{iy}}{\partial z}.$$

Fourie analyses with $\exp\left(ik_z z - i\omega t\right)$ gives the dispersion relation

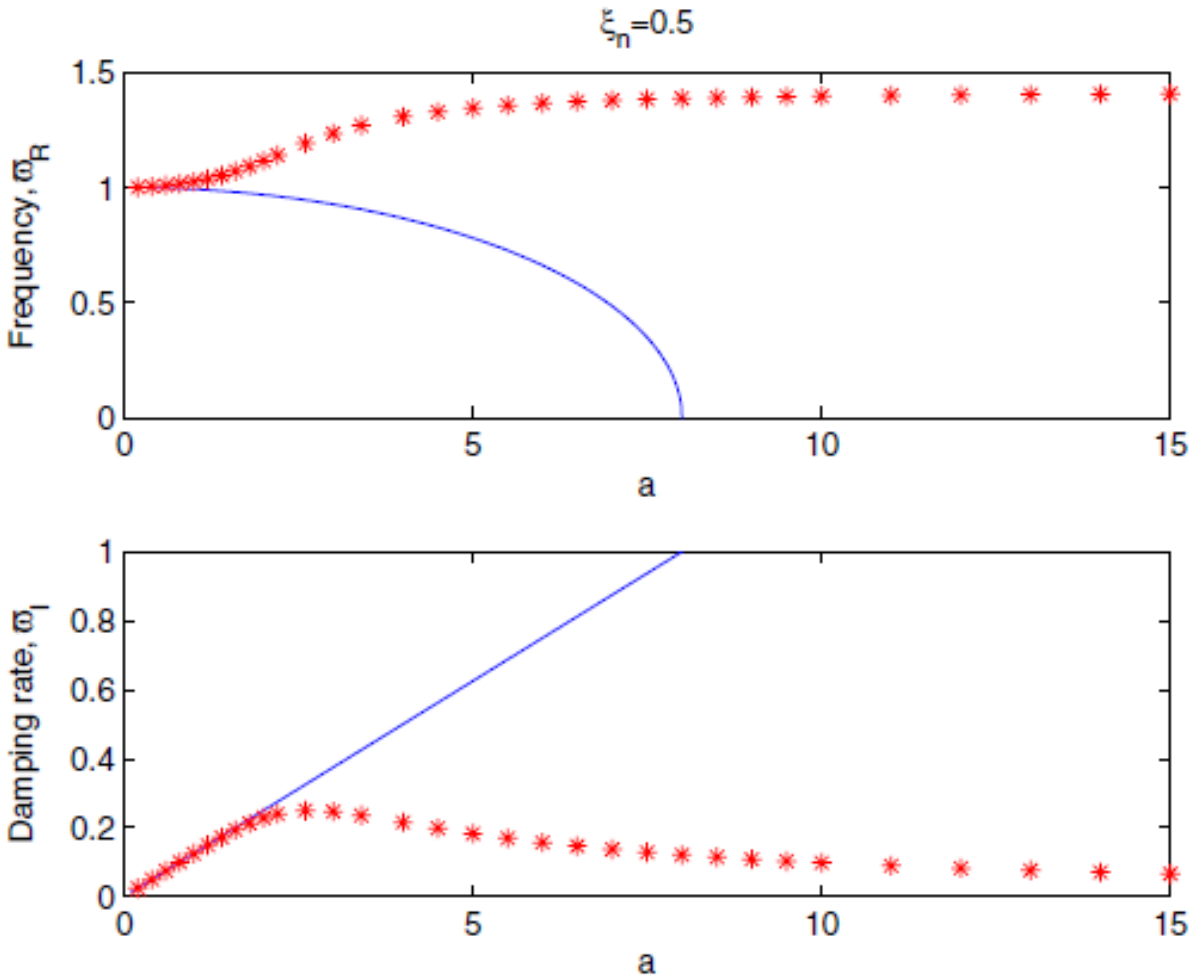
$$\omega^3 + i\frac{\alpha_{in}\rho_0}{\rho_i\rho_n}\omega^2 - k_z^2 V_A^2\frac{\rho_0}{\rho_i}\omega - ik_z^2 V_A^2\frac{\alpha_{in}\rho_0}{\rho_i\rho_n} = 0.$$

The dispersion relation has three solutions: two Alfvén waves damped by ion-neutral collision and purely imaginary solution, which is the vortex mode connected with the vorticity of neutral fluid.

The vortex mode has zero frequency in ideal fluid, but gains imaginary part when collisions between ions and neutrals are considered.

Alfvén waves in two-fluid approach

Frequency and damping rate are normalized by $k_z V_A$ and $a = \frac{k_z V_A \rho_0}{\alpha_{in}}$ is normalized Alfvén frequency.



Zaqarashvili et al. 2011

Linear magneto-acoustic waves in two-fluid approach

$$\frac{\partial \tilde{\rho}_i}{\partial t} + \rho_i \left(\frac{\partial V_{ix}}{\partial x} + \frac{\partial V_{iz}}{\partial z} \right) = 0,$$

$$\frac{\partial \tilde{\rho}_n}{\partial t} + \rho_n \left(\frac{\partial V_{nx}}{\partial x} + \frac{\partial V_{nz}}{\partial z} \right) = 0,$$

$$\frac{\partial V_{ix}}{\partial t} = -\frac{c_{si}^2}{\rho_i} \frac{\partial \tilde{\rho}_i}{\partial x} - \frac{B_z}{4\pi\rho_i} \frac{\partial b_z}{\partial x} + \frac{B_z}{4\pi\rho_i} \frac{\partial b_x}{\partial z} - \frac{\alpha_{in}}{\rho_i} \left(V_{ix} - V_{nx} \right),$$

$$\frac{\partial V_{iz}}{\partial t} = -\frac{c_{si}^2}{\rho_i} \frac{\partial \tilde{\rho}_i}{\partial z} - \frac{\alpha_{in}}{\rho_i} \left(V_{iz} - V_{nz} \right),$$

$$\frac{\partial V_{nx}}{\partial t} = -\frac{c_{sn}^2}{\rho_n} \frac{\partial \tilde{\rho}_n}{\partial x} + \frac{\alpha_{in}}{\rho_n} \left(V_{ix} - V_{nx} \right),$$

$$\frac{\partial V_{nz}}{\partial t} = -\frac{c_{sn}^2}{\rho_n} \frac{\partial \tilde{\rho}_n}{\partial z} + \frac{\alpha_{in}}{\rho_n} \left(V_{iz} - V_{nz} \right),$$

$$\frac{\partial b_x}{\partial t} = B_z \frac{\partial u_{ix}}{\partial z}.$$

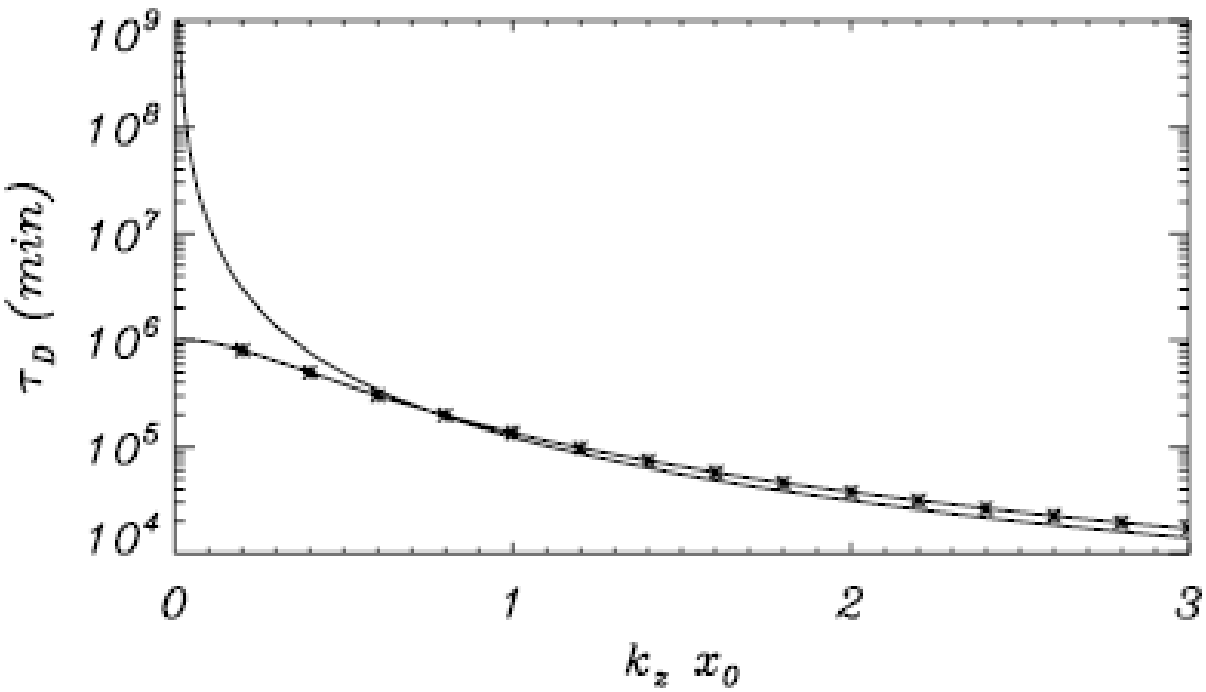
After Fourier analyses we obtain the dispersion relation for fast and slow MHD waves.

Fast waves are similar to Alfvén waves, while slow waves have different properties.

Slow waves in single-fluid approach

Damping rate of slow waves is different when it is derived from the energy equation (Braginskii 1965) and through a normal mode analysis (Forteza et al. 2007).

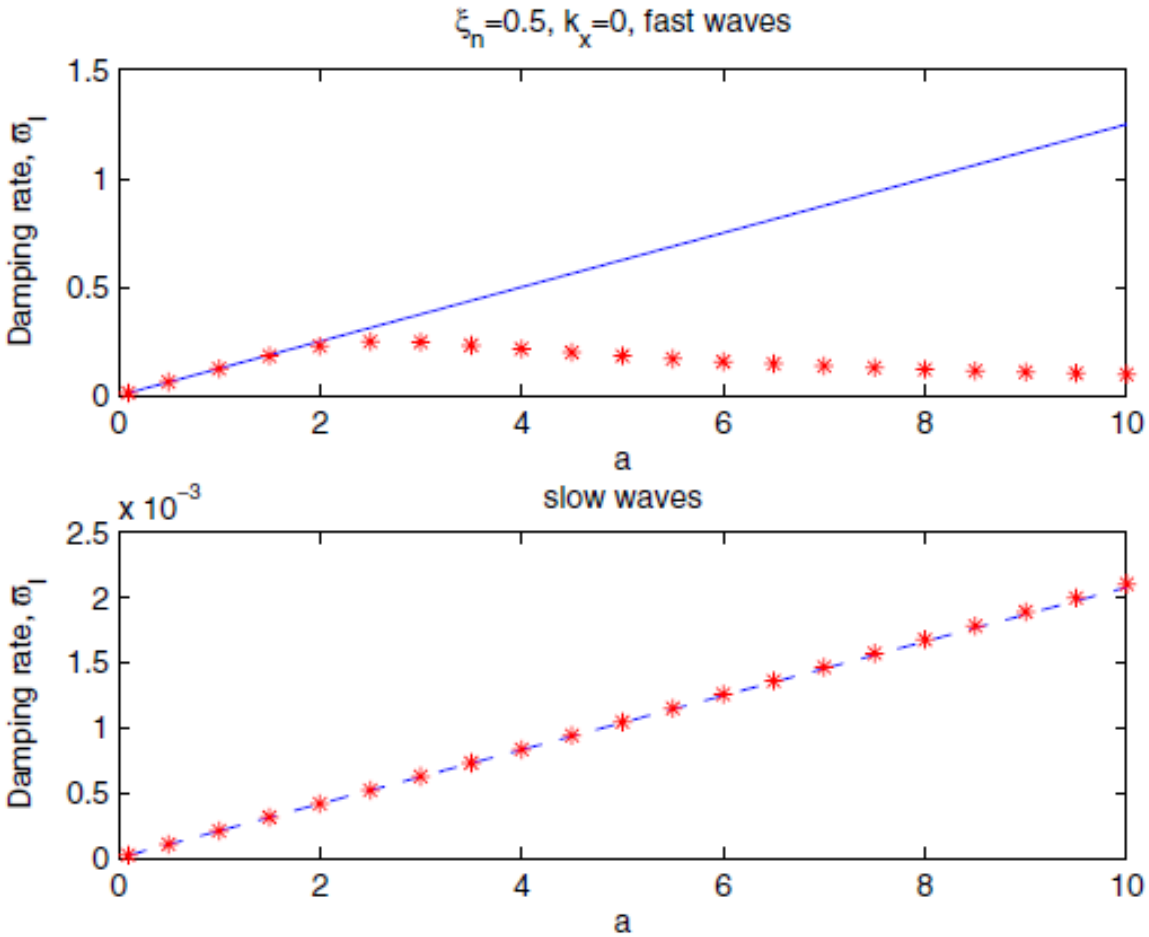
Slow magneto-acoustic waves show damping for purely parallel propagation in the case of Braginskii, while the damping is absent in Forteza et al. (2007).



Forteza et al. 2007

Slow waves in two-fluid approach

Red asterisks corresponds to two-fluid solution, dashed line corresponds to Braginskii (1965). The damping rate is zero in Forteza et al. (2007).



Zaqarashvili et al. 2011

Conclusions

- Two-fluid MHD approximation, where electron+ion gas and neutral atoms are the two components, is important when time scales approach to ion-neutral collision time;
- Single-fluid approach is a good approximation for longer times scales;
- Damping rates of Alfvén and fast magneto-acoustic waves reach their maximum near ion-neutral collision frequency, but decrease for higher frequency harmonics of the wave spectrum;
- No cut-off wave number of MHD waves appears in the two-fluid approach;
- The damping of slow magneto-acoustic waves is not zero in the parallel propagation in two-fluid approach in coincidence with Braginskii (1965);
- The two-fluid approach could be important for instability processes, when unstable harmonics have short spatial scales.

Thank you for attention!