Damping of Alfvén waves in solar partially ionized plasmas: effect of neutral helium in multi-fluid approach

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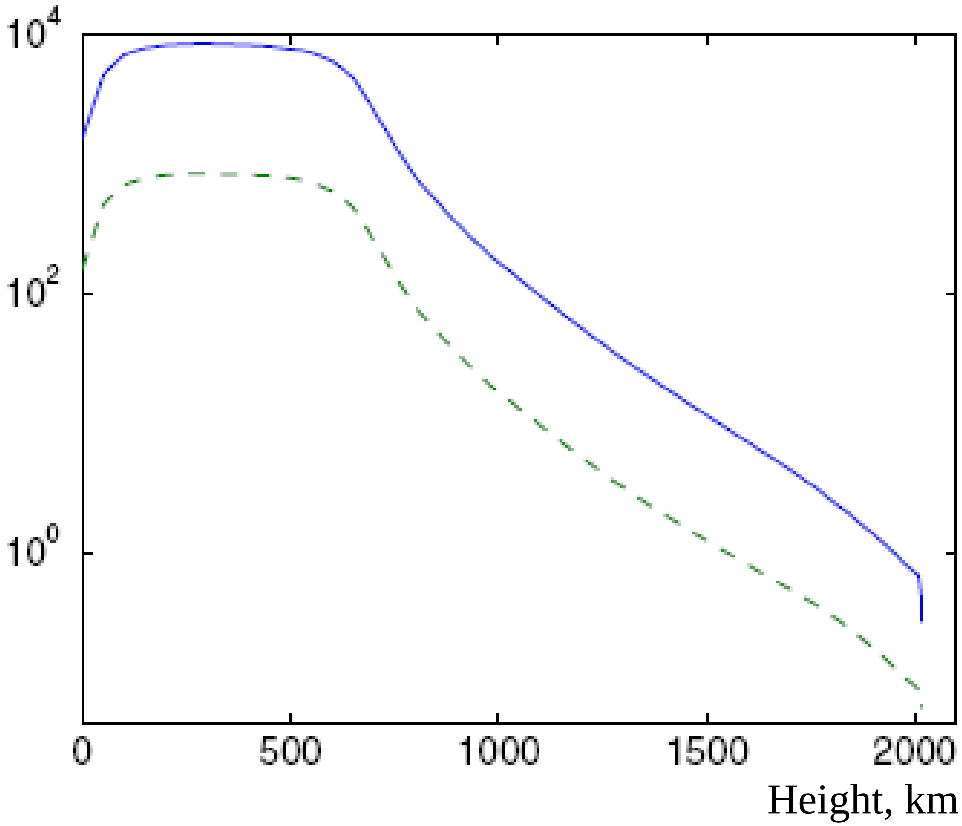
Neutral atoms in the solar atmosphere

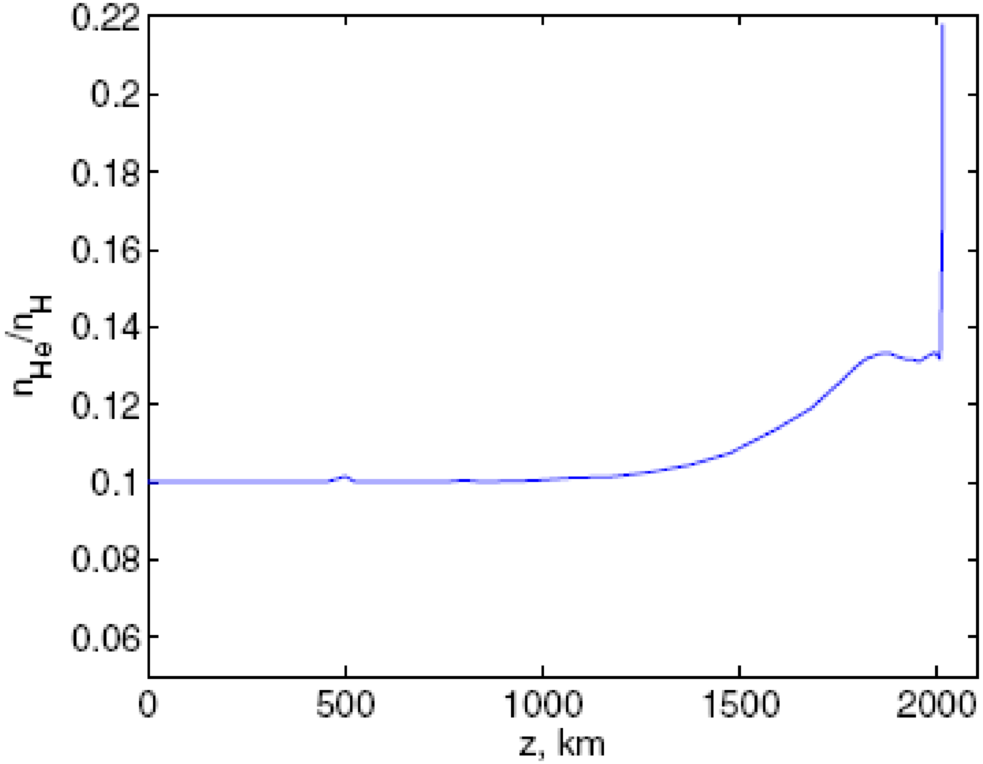


FAL93-3 model (Fontenla et al. 1993)

Blue solid line: ratio of neutral hydrogen and electron number densities.

Green dashed line: ratio of neutral helium and electron number densities.

Plasma is only weekly ionized in the photosphere, but becomes almost fully ionized in the transition region and corona.



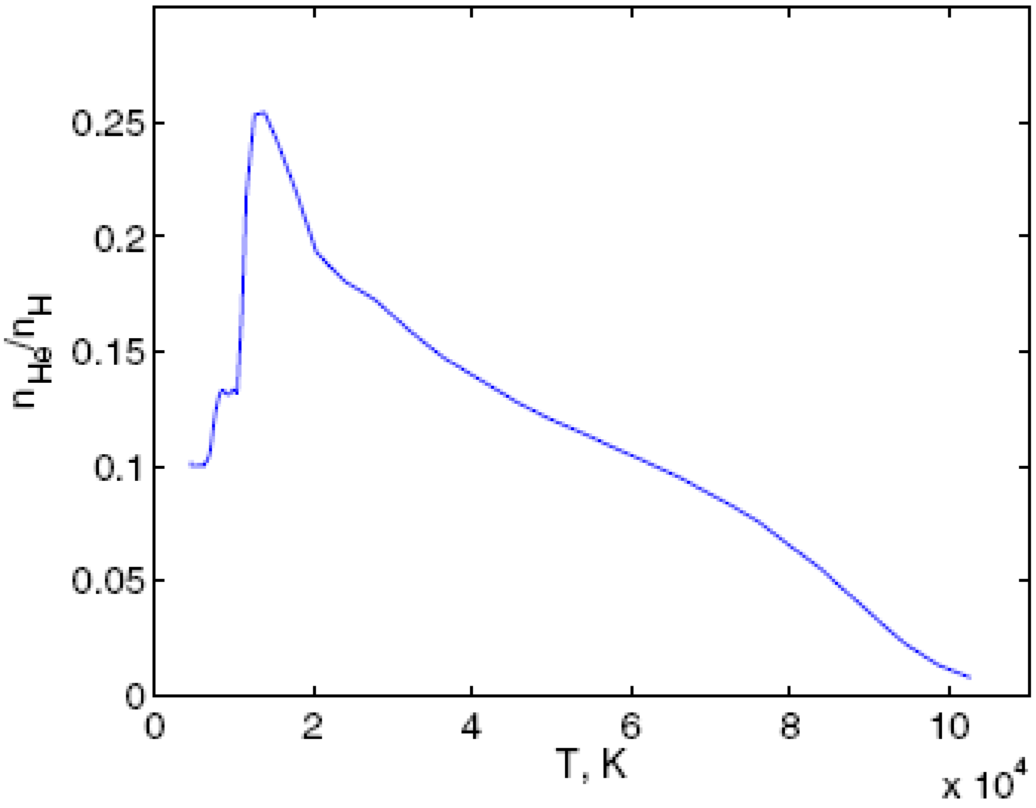
FAL93-3 model (Fontenla et al. 1993)

The ratio of neutral helium and neutral hydrogen is around 0.1 in the lower heights.

But it increases quickly up to 0.22 near chromosphere/corona transition region i.e. at 2000 km.

Neutral helium vs neutral hydrogen

The ratio of neutral helium and neutral hydrogen number densities is increased in the temperature interval 10000-40000 K.



FAL93-3 model (Fontenla et al. 1993)

Multi-fluid equations

We consider partially ionized incompressible plasma which consists of electrons, protons, singly ionized helium, neutral hydrogen and neutral helium atoms

We neglect the viscosity, the heat flux, and the heat production due to collision between particles. Then the governing equations are:

$$\nabla \cdot \vec{V}_a = 0,$$

$$m_a n_a \left(\frac{\partial \vec{V}_a}{\partial t} + \left(\vec{V}_a \cdot \nabla \right) \vec{V}_a \right) = -\nabla p_a - e_a n_a \left(\vec{E} + \frac{1}{c} \vec{V}_a \times \vec{B} \right) + \vec{R}_a,$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t},$$

$$\nabla \times \vec{B} = -\frac{4\pi}{c} \vec{j},$$

$$\vec{j} = -e \left(n_e \vec{V}_e - n_{H^+} \vec{V}_{H^+} - n_{He^+} \vec{V}_{He^+} \right).$$

Multi-fluid equations

For time scales longer than ion-electron collision time, the electron and ion gases can be considered as a single fluid. Then the five-fluid description can be changed by three-fluid description, where one component is the charged fluid (electron+protons+singly ionized helium) and other two components are the gases of neutral hydrogen and neutral helium gases.

We use the definition of total density of charged fluid

$$\rho_0 = \rho_{H^+} + \rho_{He^+}$$

and the total velocity of charged fluid as

$$\vec{V} = \frac{\rho_{H^+}\vec{V}_{H^+} + \rho_{He^+}\vec{V}_{He^+}}{\rho_{H^+} + \rho_{He^+}}.$$

The sum of momentum equations for electrons, protons and singly ionized helium is

$$\rho_0 \frac{d\vec{V}}{dt} + \rho_0 \xi_{H^+} \xi_{He^+} (\vec{w} \cdot \nabla)\vec{w} = -\nabla p + \frac{1}{c}\vec{j} \times \vec{B} + \vec{F}_t,$$

where $\vec{w} = \vec{V}_{H^+} - \vec{V}_{He^+}$ is the relative velocity of protons and helium ions.

Three-fluid equations

It can be shown that $\left|\vec{w}\right| << \left|\vec{V}\right|$ for the time scales longer than ion gyro period.

Then we obtain the three-fluid equations as

$$\rho_0 \frac{d\vec{V}}{dt} = -\nabla p + \frac{1}{4\pi}\left(\nabla \times \vec{B}\right) \times \vec{B} + \vec{F}_i,$$

$$\rho_H \frac{d\vec{V}_H}{dt} = -\nabla p_H + \vec{F}_H,$$

$$\rho_{He} \frac{d\vec{V}_{He}}{dt} = -\nabla p_{He} + \vec{F}_{He},$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{V} \times \vec{B}\right).$$

where

$$\vec{F}_i = -\left(\alpha_{H^+H} + \alpha_{H^+He} + \alpha_{He^+H} + \alpha_{He^+He}\right)\vec{V}_i + \left(\alpha_{H^+H} + \alpha_{He^+H}\right)\vec{V}_H + \left(\alpha_{H^+He} + \alpha_{He^+He}\right)\vec{V}_{He},$$

$$\vec{F}_H = -\left(\alpha_{H^+H} + \alpha_{He^+H} + \alpha_{HeH}\right)\vec{V}_H + \left(\alpha_{H^+H} + \alpha_{He^+H}\right)\vec{V}_i + \alpha_{HeH}\vec{V}_{He},$$

$$\vec{F}_{He} = -\left(\alpha_{H^+He} + \alpha_{He^+He} + \alpha_{HeH}\right)\vec{V}_{He} + \left(\alpha_{H^+He} + \alpha_{He^+He}\right)\vec{V}_i + \alpha_{HeH}\vec{V}_H.$$

Multi-fluid linear equations

We consider the wave propagation along unperturbed magnetic field, which is directed along the z axis. Then the linear Alfvén waves polarized in the y direction are governed by equations

$$\frac{\partial u_y}{\partial t} = \frac{B_z(z)}{4\pi\rho_0(z)}\frac{\partial b_y}{\partial z} - \frac{\alpha_H(z)+\alpha_{He}(z)}{\rho_0(z)}u_y + \frac{\alpha_H(z)}{\rho_0(z)}u_{Hy} + \frac{\alpha_{He}(z)}{\rho_0(z)}u_{Hey},$$

$$\frac{\partial u_{Hy}}{\partial t} = \frac{\alpha_H(z)}{\rho_H(z)}u_y - \frac{\alpha_H(z)+\alpha_{HeH}(z)}{\rho_H(z)}u_{Hy} + \frac{\alpha_{HeH}(z)}{\rho_H(z)}u_{Hey},$$

$$\frac{\partial u_{Hey}}{\partial t} = \frac{\alpha_{He}(z)}{\rho_{He}(z)}u_y - \frac{\alpha_H(z)+\alpha_{HeH}(z)}{\rho_{He}(z)}u_{Hey} + \frac{\alpha_{HeH}(z)}{\rho_{He}(z)}u_{Hy},$$

$$\frac{\partial b_y}{\partial t} = B_z(z)\frac{\partial u_y}{\partial z}.$$

Homogeneous plasma

We consider a homogeneous plasma and after Fourier transform derive the dispersion relation of Alfvén waves in the three-fluid plasma

$$\xi_H \xi_{He} a_H a_{He} \varpi^4 + i\left[\xi_H a_H (1+\xi_{He}) + \xi_{He} a_{He} (1+\xi_H)\right]\varpi^3 -$$
$$-\left[1+\xi_H + \xi_{He} + \xi_H \xi_{He} a_H a_{He}\right]\varpi^2 - i\left[\xi_H a_H + \xi_{He} a_{He}\right]\varpi + 1 = 0,$$

where

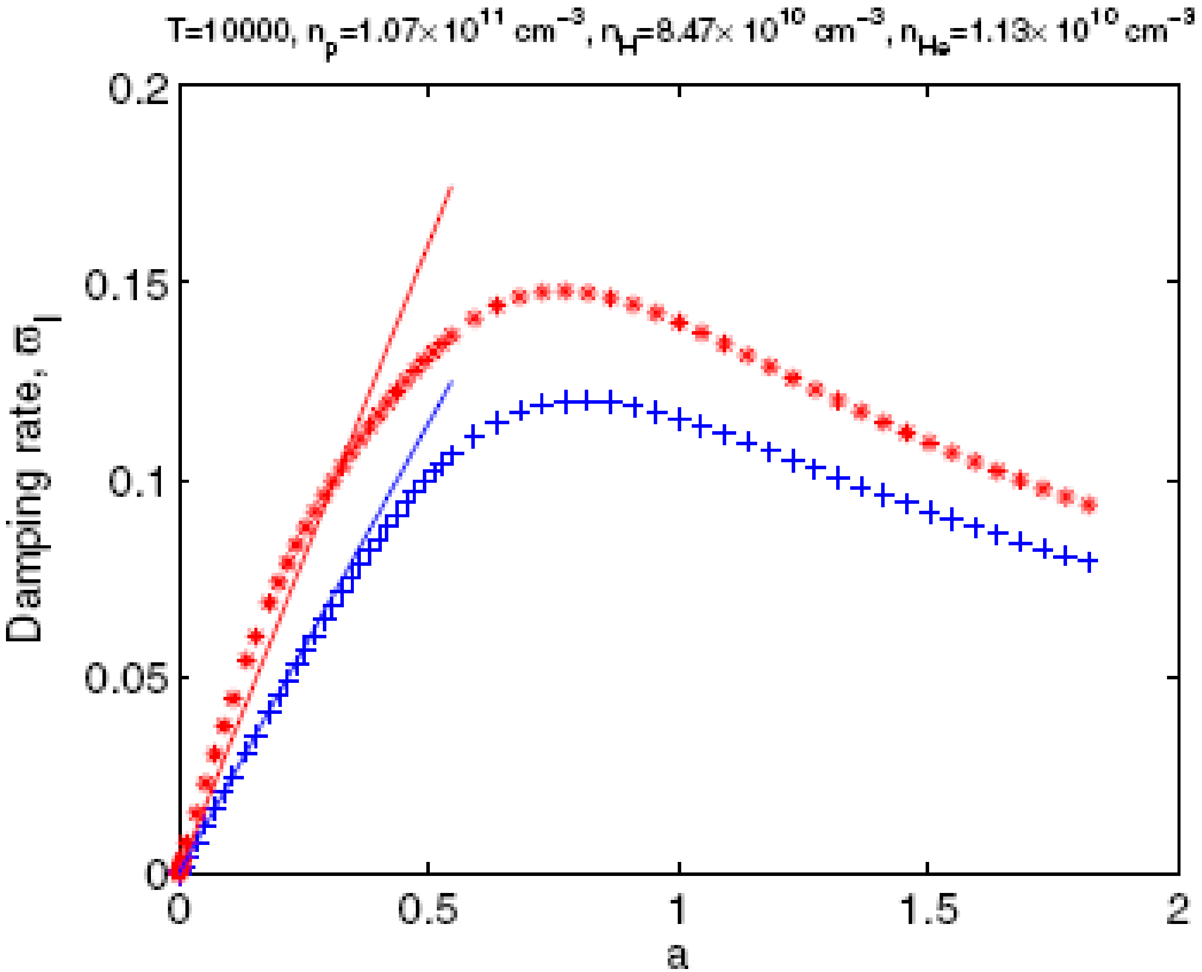
$$\varpi = \frac{\omega}{k_z v_A}, a_H = \frac{k_z v_A \rho_0}{\alpha_H}, a_{He} = \frac{k_z v_A \rho_0}{\alpha_{He}}, \xi_H = \frac{\rho_H}{\rho_0}, \xi_{He} = \frac{\rho_{He}}{\rho_0}.$$

The dispersion relation has four different roots: the two complex solutions, which correspond to Alfvén waves damped by ion-neutral collision and two purely imaginary solutions, which correspond to damped vortex solutions of neutral hydrogen and neutral helium fluids.

We consider only Alfvén waves.

Upper chromosphere

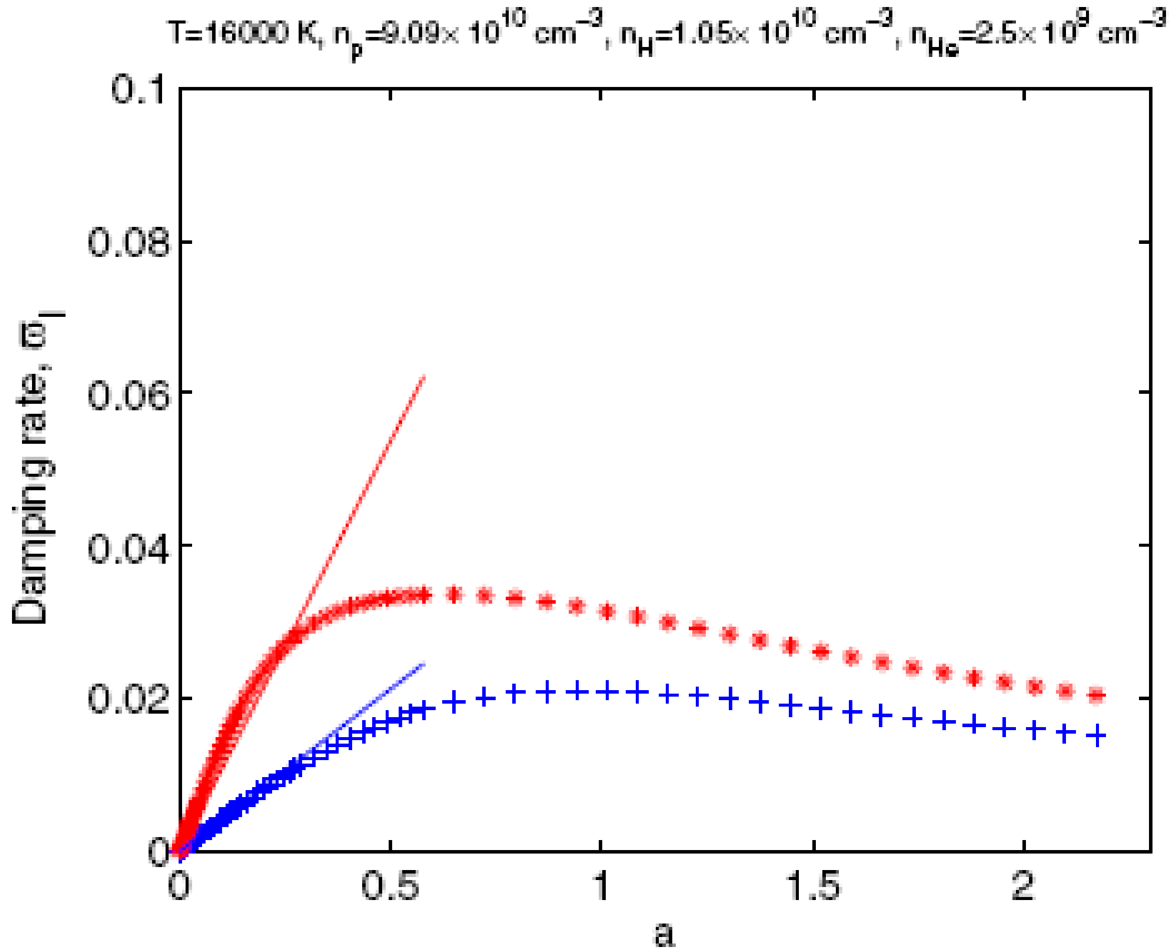
Chromosphere: 1995 km height above the photosphere.



Zaqarashvili et al. 2011

Upper chromosphere

Chromosphere: 2015 km height above the photosphere.



Zaqarashvili et al. 2011

Collision frequencies

Mean ion-neutral collision frequency is (Zaqarashvili et al. 2011)

$$\nu_{in} = \alpha_{in}\left(\frac{1}{m_i n_i} + \frac{1}{m_n n_n}\right) = 2(n_i + n_n)\sigma_{in}\sqrt{\frac{kT}{\pi m_i}}.$$

The collision frequency is very high in the photosphere, but decreases upwards.

The collision frequency between protons and neutral hydrogen atoms estimated from FAL93-3 model can be estimated as

	z=0:	z=900:	z=1900:
ν_{in}	$=8.6\ 10^{6}$ Hz	$6.2\ 10^{3}$ Hz	24 Hz

This means that the Alfvén waves with periods > 1 s can be easily considered in the single-fluid approach.

Alfvén waves in single-fluid MHD

We consider the total density

$$\rho = \rho_0 + \rho_H + \rho_{He},$$

total velocity

$$V_y = \frac{\rho_0 u_y + \rho_H u_{Hy} + \rho_{He} u_{Hey}}{\rho_0 + \rho_H + \rho_{He}},$$

relative velocity between ions and neutral hydrogen

$$w_H = u_y - u_{Hy}$$

and relative velocity between ions and neutral helium

$$w_{He} = u_y - u_{Hey}.$$

Then we find that

$$u_y = V_y + \xi_H w_H + \xi_{He} w_{He}.$$

Alfvén waves in single-fluid MHD

Consecutive subtractions of multi-fluid equations and neglect of inertial terms leads to the equations

$$w_H = \frac{B_z}{4\pi}\left[\frac{\alpha_{He}}{\alpha}\xi_H + \frac{\alpha_{HeH}}{\alpha}(\xi_H + \xi_{He})\right]\frac{\partial b_y}{\partial z},$$

$$w_{He} = \frac{B_z}{4\pi}\left[\frac{\alpha_H}{\alpha}\xi_{He} + \frac{\alpha_{HeH}}{\alpha}(\xi_H + \xi_{He})\right]\frac{\partial b_y}{\partial z},$$

where $\alpha = \alpha_H\alpha_{He} + \alpha_H\alpha_{HeH} + \alpha_{He}\alpha_{HeH}.$

Then the sum of multi-fluid equations leads to the single-fluid equations

$$\frac{\partial V_y}{\partial t} = \frac{B_z(z)}{4\pi\rho(z)}\frac{\partial b_y}{\partial z},$$

$$\frac{\partial b_y}{\partial t} = B_z(z)\frac{\partial V_y}{\partial z} + B_z(z)\frac{\partial}{\partial z}\left(\frac{\eta_c(z)}{B_z(z)}\frac{\partial b_y}{\partial z}\right)$$

where

$$\eta_c = \frac{B_z^2(z)}{4\pi}\left(\frac{\alpha_{He}(z)}{\alpha(z)}\xi_H^2(z) + \frac{\alpha_H(z)}{\alpha(z)}\xi_{He}^2(z) + \frac{\alpha_{HeH}(z)}{\alpha(z)}\left(\xi_H(z) + \xi_{He}(z)\right)^2\right)$$

is the coefficient of Cowling diffusion.

Dispersion relation

From these two equations we get

$$\frac{\partial^2 V_y}{\partial t^2} = \frac{B_z(z)}{4\pi\rho(z)}\frac{\partial}{\partial z}\left[B_z(z)\frac{\partial V_y}{\partial z} + B_z(z)\frac{\partial}{\partial z}\left(\frac{\eta_c(z)}{V_A^2(z)}\frac{\partial V_y}{\partial t}\right)\right].$$

For homogeneous atmosphere we have

$$\omega^2 + i\eta_c k_z^2\omega - k_z^2 V_A^2 = 0$$

which has two complex solutions

$$\omega = \pm k_z V_A\sqrt{1-\frac{\eta_c^2 k_z^2}{4V_A^2}} - i\frac{\eta_c k_z^2}{2}.$$

Real part of the complex frequency gives cut-off wave number

$$k_z = \pm\frac{2V_A}{\eta_c}.$$

Dispersion relation

Normalized damping rate is

$$\left|\tilde{\omega}_i\right| = \left|\frac{\omega_i}{k_z V_A}\right| = \frac{1}{2}\frac{k_z V_A}{\rho}\frac{\alpha_{He}\rho_H^2 + \alpha_H \rho_{He}^2 + \alpha_{HeH}\left(\rho_H + \rho_{He}\right)^2}{\alpha_H \alpha_{He} + \alpha_H \alpha_{HeH} + \alpha_{He}\alpha_{HeH}}.$$

In the low chromosphere , where plasma is only weakly ionized, we have $\alpha_{HeH} >> \alpha_H, \alpha_{He}$ therefore

$$\left|\tilde{\omega}_i\right| = \frac{1}{2}\frac{k_z V_A}{\rho}\frac{\left(\rho_H + \rho_{He}\right)^2}{\alpha_H + \alpha_{He}}.$$

This expression was used by De Pontieu et al. (2001) and Soler et al. (2010).

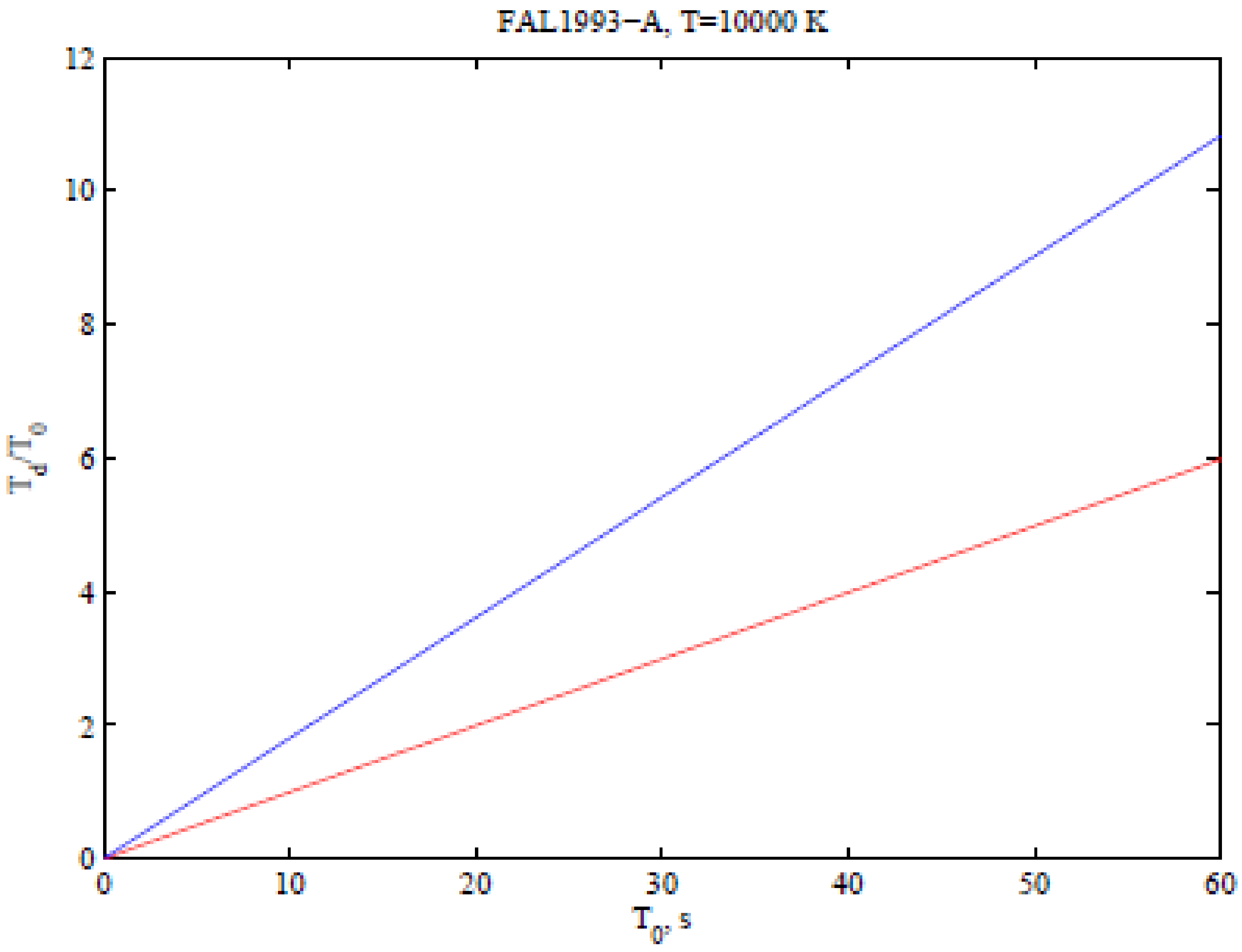
On the other hand, in higher regions of the chromosphere, where $\alpha_{HeH} << \alpha_H, \alpha_{He}$, we have

$$\left|\tilde{\omega}_i\right| = \frac{1}{2}\frac{k_z V_A}{\rho}\left[\frac{\rho_H^2}{\alpha_H} + \frac{\rho_{He}^2}{\alpha_{He}}\right].$$

In the middle chromosphere, spicules and prominences the general expression should be used.

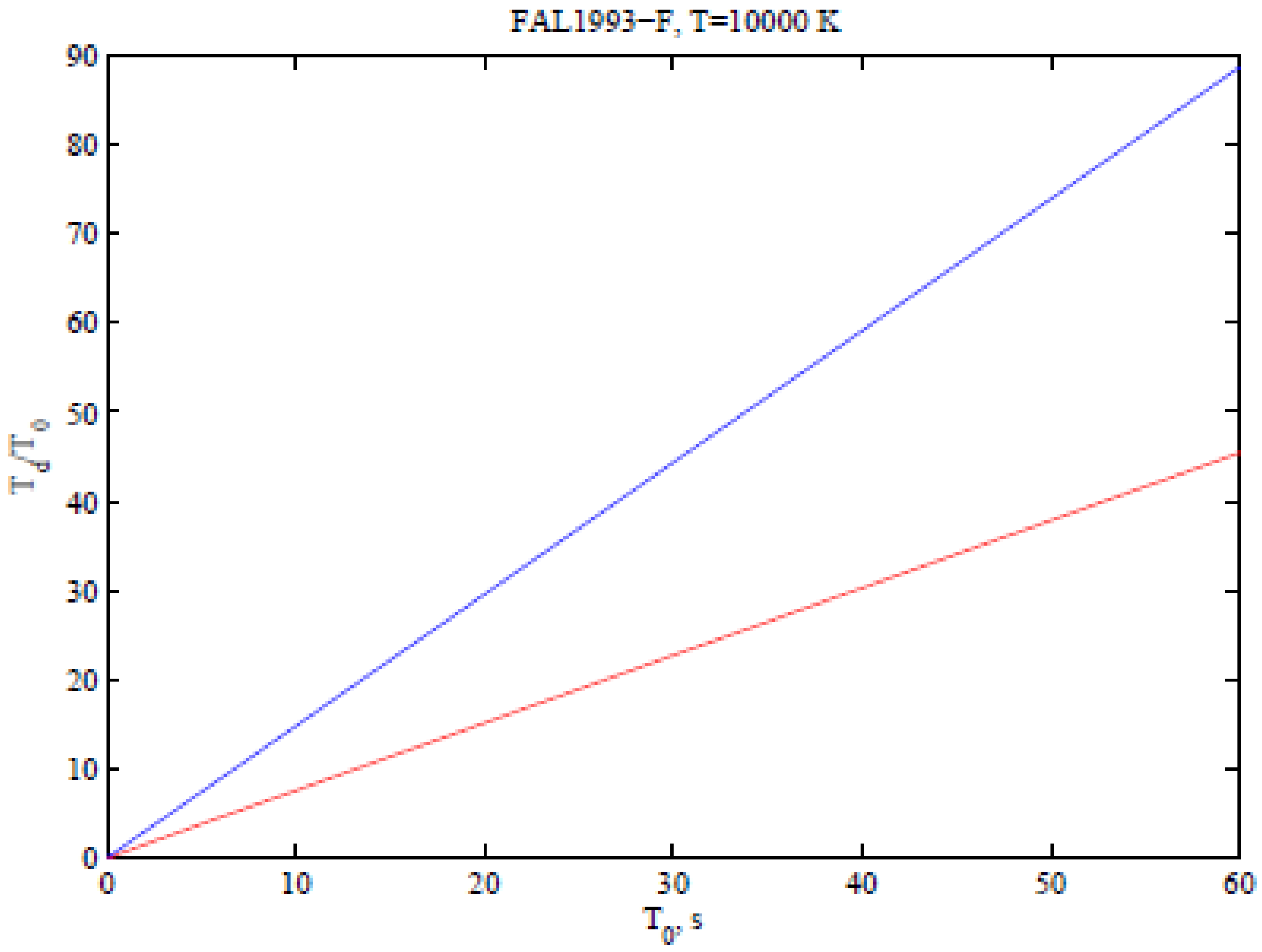
Faint cell center area (FAL93-A)

$n_i = 1.55\ 10^{10}\ cm^{-3}$, $n_H = 1.71\ 10^{10}\ cm^{-3}$, $n_{He} = 2.31\ 10^{9}\ cm^{-3}$



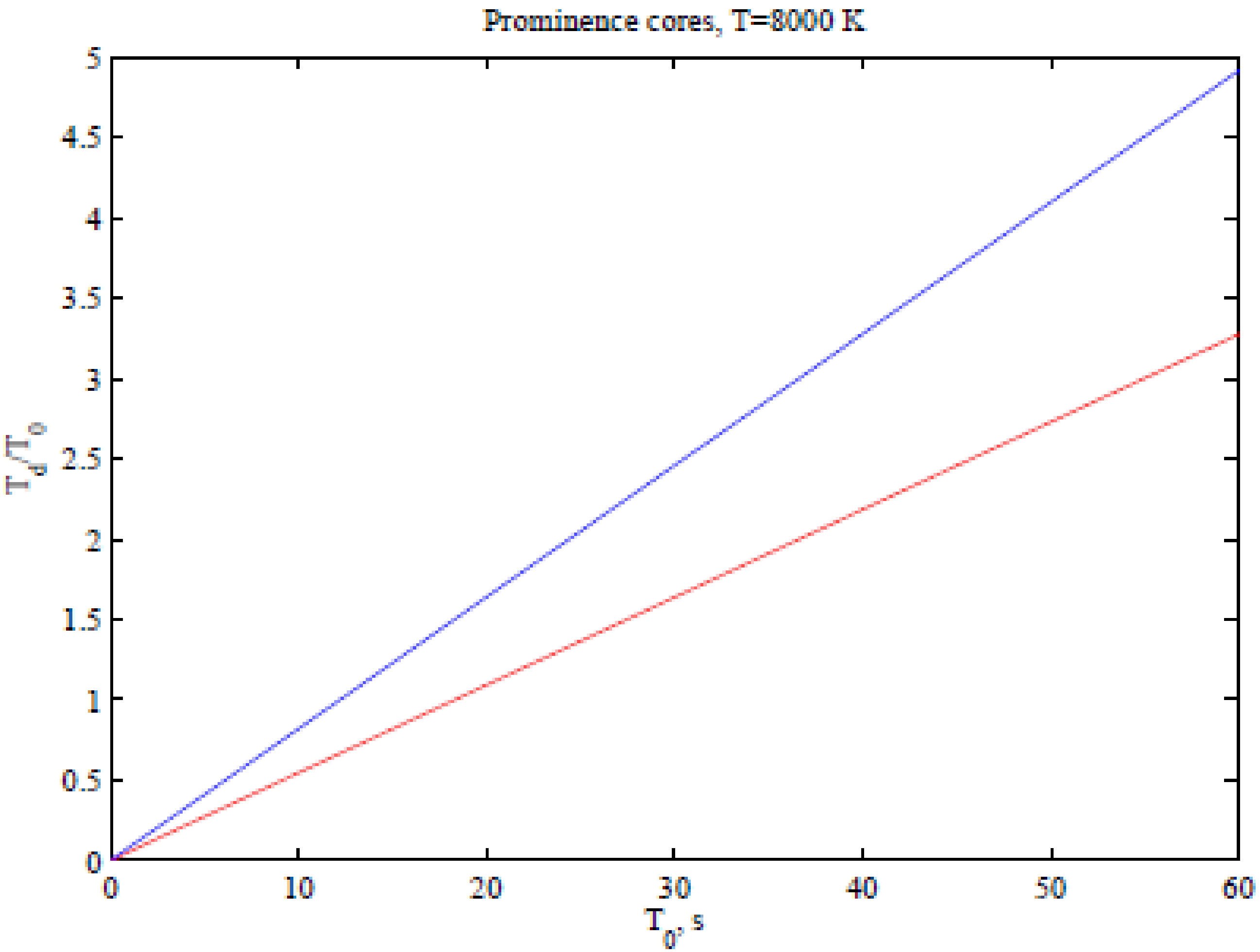
Bright network (FAL93-F)

$n_i = 1.07\ 10^{11}\ cm^{-3}$, $n_H = 8.47\ 10^{10}\ cm^{-3}$, $n_{He} = 1.19\ 10^{9}\ cm^{-3}$



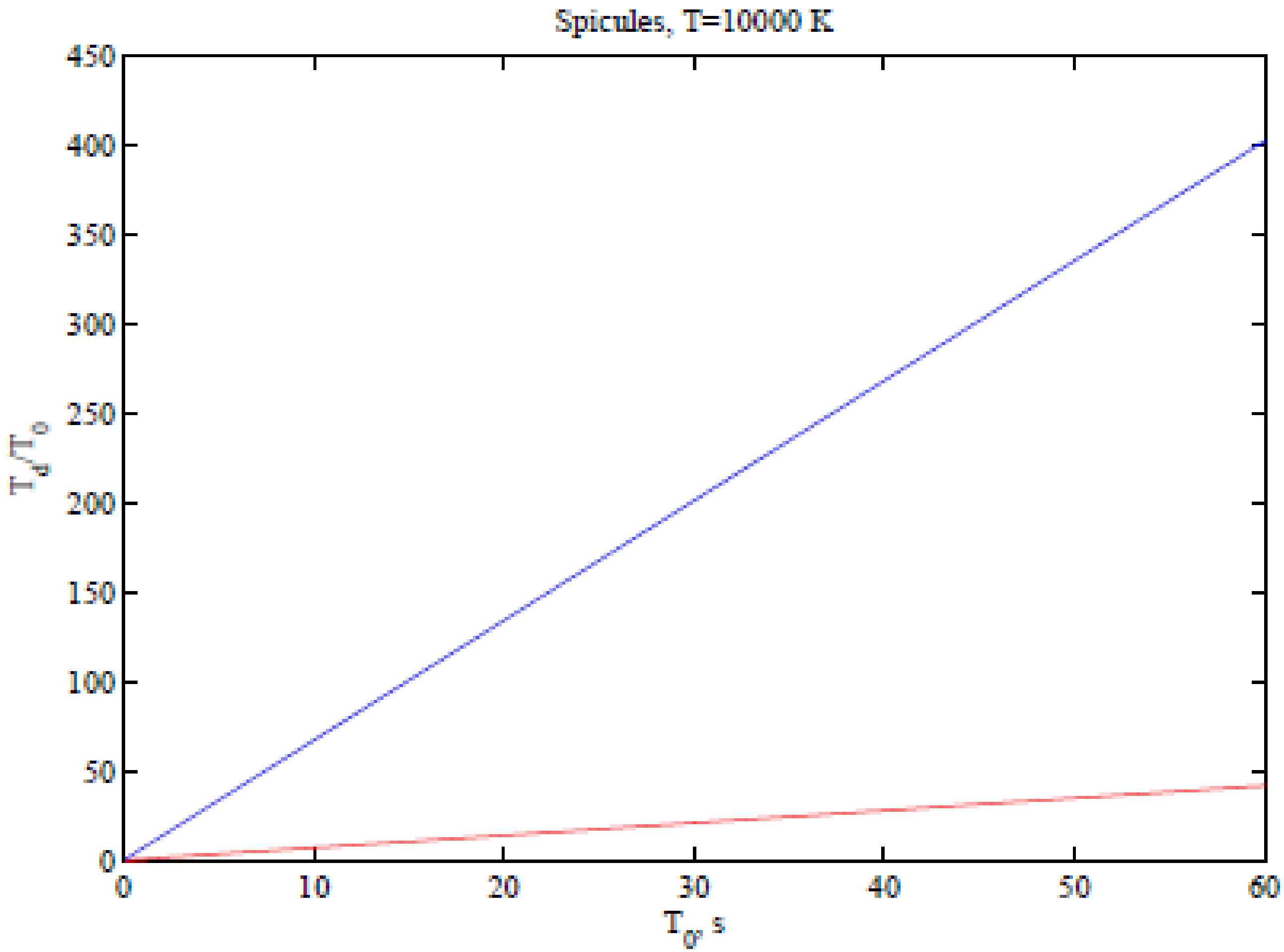
Prominence cores

$$n_i = 10^{10}\ cm^{-3},\ n_H = 2\ 10^{10}\ cm^{-3},\ n_{He} = 2\ 10^{9}\ cm^{-3}$$



Spicules

$$n_i = 10^{11}\ cm^{-3},\ n_H = 10^{10}\ cm^{-3},\ n_{He} = 10^{10}\ cm^{-3}$$



Conclusions

- The ratio of neutral helium and neutral hydrogen number densities is increased for T=10000-40000 K.
- Consequently, neutral helium atoms significantly enhance the damping of Alfvén waves in the chromospheric, spicule and prominence plasma for T=8000-40000 K.
- The multi-fluid approach reveals that the damping rate is maximal near ion-neutral collision frequency and then decreases for higher harmonics.
- The single-fluid approach is valid for the Alfvén waves with longer period (> 1 s).
- The expression of damping rate, which has been frequently used, is only valid for weakly ionized plasma.
- The modified expression of damping rate should be used in higher chromosphere, spicules and prominences.

Thank you for your attention!