Population synthesis models
From stellar evolution models to synthetic populations in the Milky Way

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Main goals:

- what determines the star counts across the Milky Way, as a function of magnitude and wavelength?
- how to model it? how to extract information from the model–data comparisons?

Stars we see in the MW are the result of:

1. stellar formation
2. stellar evolution
3. stellar dynamics
Introduction

Scheme

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Stars we see in the MW are the result of:

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2. stellar evolution
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Five lectures:

1. Overview of stellar evolution
2. Isochrones, modeling star clusters and binaries
3. Modeling external galaxies
4. Modeling the Milky Way
5. Recent problems and opportunities
Lecture goals:

What determines stellar $L$, $T_{\text{eff}}$, surface composition and lifetimes?

- overview of stellar evolution
  - evolution for different masses/metallicities
  - only main evolutionary phases – either long-lived or bright
  - emphasis on low- and intermediate-mass – the only to tell something about galaxy story
  - changes in surface composition
Basic references

- **textbooks**
  - Cox & Giuli, *Principles of stellar structure*
  - Hansen & Kawaler, *Stellar interiors*
  - Kippenhahn & Weigert, *Stellar structure and evolution*
  - de Loore & Doom, *Structure and evolution of single and binary stars*
  - Salaris & Cassisi *Evolution of stars and stellar populations*
  - Habing & Olofsson (eds.), *AGB stars*

- **reviews**
  - Chiosi, Bertelli & Bressan 1992, ARA&A (the HR diagram)
  - Renzini & Fusi Pecci 1988, ARA&A (globular cluster stars)
  - Salaris, Cassisi & Weiss 2002, PASP (the RGB)
  - Herwig 2005, ARA&A (the AGB)
Stellar evolution models

\[
\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad \text{mass continuity (1)}
\]

\[
\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2 \rho} \frac{\partial^2 r}{\partial t^2} \quad \text{hydrostatic equilibrium (2)}
\]

\[
\frac{\partial l}{\partial m} = \epsilon_n - \epsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho} \frac{\partial P}{\partial t} \quad \text{energy balance (3)}
\]

\[
\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla \quad \text{transport equation (4)}
\]

\[
\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho} \left( \sum_j r_{ji} - \sum_k r_{ik} \right) \quad \text{nuclear burning (+ mixing) (5)}
\]

with the Schwarzschild criterion (\(\nabla_{\text{rad}} > \nabla_{\text{ad}}\) for convection), and the mixing length theory

\[
\nabla = \frac{d \ln T}{d \ln P} = \begin{cases} 
\approx \nabla_{\text{ad}} & \text{for deep convective regions} \\
> \nabla_{\text{ad}} & \text{for external convective regions} \\
\nabla_{\text{rad}} = \frac{3}{16\pi acG} \frac{\kappa lP}{mT^4} & \text{for radiative regions}
\end{cases}
\]
Main uncertainties

critical data:

- **Rosseland mean opacities** $\kappa$: determine $\nabla$ (radiative and super-adiabatic), extension of convective zones, position of Hayashi line well known for high $T$ (complete ionization, $e^-$ scattering, $e^-$ conduction) modest uncertainties persist for $\sim 10^7$ K (partial ionization), and for cool giant envelopes (molecules).

- **reaction rates** $r_{ij}$: despite great uncertainties, are generally not critical due to the thermostat effect in burning regions main exception: $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ affects duration of CHeB phase and extension of its blue loop (among others)

- **plasma neutrino losses** $\epsilon_\nu$: determines cooling of degenerate cores, affects onset of He-flash
Main uncertainties

critical approximations:

- **boundary criteria for convective zones** (classical versus overshooting): affect MS lifetime and luminosity of all post-MS phases
- **semiconvection + breathing pulses**: affects CHeB phase if overshooting not efficient
- **the mixing-length theory**: $\alpha_{\text{ml}}$ is a free parameter, tuned to fit Sun or RGB
- **mass loss (RGB tip and AGB)**: no good theory, just “reasonable prescriptions”
- **persistent problems in modeling 3rd dredge-up on TP-AGB**: not clear if an issue of numerical accuracy or treatment of convection

non-critical approximations:

- **microscopic diffusion**: effect is cancelled by 1st dredge-up
- **grey atmosphere approximation**: effect detectable in the Sun, minor for most other stars
Databases of stellar tracks and isochrones

Many different databases exist. While input physics is becoming ~ the same for all sets, default assumptions vary a lot.

**Geneva**: concentrates on massive stars, with moderate overshooting and rotation

**Padova**: (now Padova-Trieste) complete database for population synthesis, adopts moderate overshooting, includes TP-AGB phase

**Pisa**: mainly low- and intermediate-mass stars, with pre-MS and diffusion for low-mass stars

**Yale-Yonsei**: extended isochrone sets, both classical and overshooting models, lacks CHeB and later phases

**BaSTI (Teramo)**: also aimed at population synthesis, extended in $[\text{M/H}]$, includes $\alpha$-enhanced compositions

**FRUITY (Teramo)**: with a focus on nucleosynthesis, esp. massive stars and many others
Databases of stellar tracks and isochrones

example: Padova tracks and isochrones, $Z = 0.008$ solar-scaled, (Bertelli et al. 1994 + Girardi et al. 2000), http://stev.oapd.inaf.it/
The way out of main sequence

- if $M \leq 1 \ M_\odot$,
  - *pp* chain, extended radiative burning region
  - as MS goes on, result is a continuous $X$ profile
- if $M > 1 \ M_\odot$,
  - CNO-cycle, convective burning region
  - fraction of convective region increases with mass and overshooting efficiency
  - anyway, its size decrease with time in the MS

In both cases, at the MS termination a continuous $X$ profile remains

- Core shrinking, envelope expansion and inward penetration of external convection: it reaches the $X$ profile and the first dredge-up takes place ($\Delta Y \lesssim 0.03$).
- The H-Burning shell (CNO cycle) starts to migrate outward along this profile.
To RGB or not to RGB?

The H-exhausted core has no source of nuclear energy: it must shrink, and heatens due to its own gravitational energy (virial theorem). Either

1. the core becomes $e^{-}$-degenerate before He-ignition, and nearly isothermal due to the high $e^{-}$-conduction. $\log T_{\text{core}} \sim 7.5$ ($\sim$ bottom of H-burning shell), and He-ignition is prevented unless the core grows. The star becomes a RGB; or
2. the core heatens enough for He-ignition ($10^8$ K) before $e^{-}$-degeneracy occurs. This causes expansion of the core, preventing further $e^{-}$-degeneracy. The RGB is failed.

Transition mass

$$M_{\text{HeF}} = \begin{cases} 2.4 \, M_{\odot} & \text{for classical models} \\ 2.0 \, M_{\odot} & \text{for moderate overshoot} \end{cases}$$  \hspace{1cm} (7)$$

$$\rho_c \times T_c \ (\text{Iben 1991})$$

Core mass at He ignition (Girardi 2000)
The RGB evolution

- RGB starts with a H-burning shell as thick as 0.1 $M_\odot$
- $T_{\text{shell}}$ is high and requires CNO cycle, $pp$ chain works externally but is negligible as energy source
- the H-burning shell soon becomes very thin, of just some 0.001 $M_\odot$

The mass structure of a RGB star:

1. $\epsilon^-$-degenerate ($\sim$ isothermal) He core of small radius and increasing mass $M_{\text{core}} \gtrsim 0.3 - 0.5 ~ M_\odot$
2. very thin H-burning shell
3. very thin radiative region with steep $P$ gradient
4. convective envelope of mass $M_{\text{env}} \sim M - M_{\text{core}}$ and mean molecular weight

$$\mu \simeq \frac{4}{5X + 3}$$

- Under these circumstances, the RGB star must
  - locate close to its Hayashi line
  - follow a core mass – luminosity relation.
The Hayashi line

Locus of completely convective star of $M, X_i$.

- $\nabla = \nabla_{\text{conv}}$
- let’s suppose $\nabla_{\text{conv}} = \nabla_{\text{ad}}$
  - if star has $\nabla > \nabla_{\text{ad}}$ moves to the right of Hayashi line
  - in mixing-length theory, $\nu_{\text{conv}} \sim (\nabla - \nabla_{\text{ad}})^{1/2}$
  - large $\nu_{\text{conv}} \rightarrow \nabla \sim \nabla_{\text{ad}}$ and back to Hayashi line
The core mass – luminosity relation(s)

- Paczyński 1970: discovery in static AGB models, \( L/L_\odot = 5.92 \times 10^4 (M/M_\odot - 0.52) \)
- Reifsdal & Weigert 1970, Kippenhahn 1981: derivation via homology relations (\( r/R_{\text{core}} = r'/R'_{\text{core}} \)) find basic dependencies, \( L \propto (M_{\text{core}} \mu)^7 \) for RGB
- Boothroyd & Sackmann 1988: complete AGB models, find dependence on \( \mu^3 \)
- Marigo 2000: unified formalism with all main dependencies

The CMLR relation says:

Luminosity of shell-burning stars with degenerate cores is determined by their core mass, and insensitive to the envelope mass

- Conditions for validity are (Tuchman et al. 1983, Marigo et al. 1999):
  - a degenerate core
  - surrounded by a narrow radiative burning shell with \( L \approx L_{\text{shell}} \)
  - a thin (\( \Delta M \ll M_{\text{core}} \)) radiative buffer with steep \( T, P \) gradients
  - and a convective envelope

- In fact, CMLRs apply to:
  - RGB stars (all)
  - AGB stars (only during quiescent interpulse phases, if without HBB)
  - PNe central stars (as long as the H-shell is still burning)
  - novae (in their stationary burning phases)
RGB – LF and bump

- Thomas (1967), Iben (1968):
  - H-shell meets H-discontinuity left by 1st dredge-up
  - luminosity temporarily drops, why? $\rightarrow L \propto \mu^7$

- RGB-bump position depends on age, metallicity, and convection criterion, see e.g. Cassisi & Salaris (1997) for $\Delta V_{\text{HB}}^{\text{bump}}$

- after bump $\Delta \log N/\Delta \log L \sim \log L + \text{constant}$, why? $\rightarrow L \propto M_{\text{core}}^7$, $\frac{dM_{\text{core}}}{dt} \propto L/(qX)$ $\rightarrow$ (evolutionary stratigraphy, Sandage 1957)

Girardi et al. (2000) track

47 Tuc (Renzini & Fusi Pecci 1988)
He-flash

- along RGB, core
  - heatens due to accretion of mass left over by shell
  - cools due to plasma neutrino losses
- at $\log L/L_\odot$, $M_{\text{core}} \simeq 0.47 \ M_\odot$, off-center He-ignition
- He-flash lifts degeneracy in few $10^4$ yr, star settles in CHeB phase

Thomas 1967:
Stellar evolution

From MS to He-ignition

**RGB tip**

- depends on $M_{\text{core}}$ at flash $\rightarrow$ plasma neutrino losses, opacities and equation of state can change it by $\sim 0.01 \, M_\odot$ or $\Delta M_{\text{bol,tip}} \sim 0.1$ mag
- for old metal-poor populations $L_{\text{tip}} \sim$ constant and at lower $M_I$ than for intermediate-age metal-rich $\rightarrow$ good standard candle
- theoretical and empirical calibration: see Salaris et al. (2002)

Girardi et al. (2000) $Z = 0.001$ tracks

$Z = 0.019$
Core helium burning – basic facts

- $^4\text{He}(\alpha, \gamma)^8\text{Be}(\alpha, \gamma)^{12}\text{C}$, $T_c \sim 10^8$ K, $\epsilon_{3\alpha} \propto T^{-40}$ → convective core
- anyway, H-burning shell accounts for $> 1/2$ of total $L$
- when $Y_c \rightarrow 0$, $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ operates (but rates uncertain by factor of $\sim 3$), and determines final $(C/O)_{\text{core}}$
- opacity of C+O mix grows with $X_{\text{CO}}$
  - core grows with time, and $\nabla$ becomes discontinuous at border → high convective velocities
  - semi-convective mixing + breathing pulses at later stages (Schwarzschild 1971, Faulkner & Robertson 1972, Castellani et al. 1985)
- if overshooting is efficient, it is the dominant effect (see Chiosi et al. 1992)
- lifetimes of CHeB are affected

Cannon & Faulkner (1973)
Stellar evolution
From CHeB to TP-AGB

CHeB in the HR diagram

- $L_{\text{He}}$ closely follows $M_{\text{He}}$ at He-ignition
  - for intermediate-mass stars: $M_{\text{He}} \propto M$ and hence $L_{\text{He}}$ increases with $M$, $t_{\text{He}}$ decreases
  - for low-mass stars: $M_{\text{He}} \simeq 0.47 M_\odot$ and hence $L_{\text{He}} \simeq$ constant, $t_{\text{He}} \simeq 10^8$ yr
  - for transition mass $M_{\text{HeF}}$: $M_{\text{He}}$ has minimum of 0.33 $M_\odot$ and therefore also $L_{\text{He}}$, $t_{\text{He}}$ has a maximum

- $L_H$ in general increases with $M_{\text{env}}$

- $T_{\text{eff}}$:
  - intermediate-mass stars: starts close to Hayashi line, may Cepheid loop when $L_H > L_{\text{He}}$, most easily at low $Z$
  - low-mass stars: mostly cool (red clump), but become hot if $M_{\text{env}} \to 0$ (HB)
The low-mass CHeB: red clump or HB

- $T_{\text{eff}}$ determined mostly by $Z$ and $M_{\text{env}}$
- to left of RR lyrae instability strip if high age (low $M$), low $Z$, or high $\dot{M}$
To AGB or not to AGB?

The He-exhausted core has no source of nuclear energy: it must shrink, and heatens due to its own gravitational energy (virial theorem). Either

1. the core becomes $e^-$-degenerate before C-ignition, and nearly isothermal due to the high $e^-$-conduction. $\log T_{\text{core}} \sim 8.0$ ($\sim$ bottom of He-burning shell), and C-ignition is prevented unless the core grows. The star becomes an AGB; or

2. the core heatens enough for C-ignition ($\sim 5 \times 10^8$ K) before $e^-$-degeneracy occurs. This causes expansion of the core, preventing further degeneracy. The AGB is failed.

Transition mass

$$M_{\text{up}} = \begin{cases} 9 \, M_\odot & \text{for classical models} \\ 6 \, M_\odot & \text{for moderate overshoot} \end{cases}$$ (8)

Core mass at beginning of TP-AGB (Girardi 2000)

$$\rho_c \times T_c$$ (Iben 1991)
Early AGB

- phase of He-shell formation and thinning, similar to subgiant branch
- a temporary increase of $L_{\text{He}}$ is accompanied by reduction of $L_{\text{H}}$
- expansion of envelope and deepening of convection
- in stars $M \gtrsim 4M_\odot$, $L_{\text{H}} \rightarrow 0$ (H-shell extinguishes) and second dredge-up occurs
- at end of E-AGB, He-shell approaches H-shell and reignites it $\rightarrow$ double-shell phase actually begins
The early AGB clump

- In low-mass stars, E-AGB luminosity first drop, then increase again, during main phase of shell thinning.
- Lifetime of $\sim 10^7$ year, creates clump with $N_{EAGB} \sim 0.1 N_{CHeB}$, about 1 mag above clump/HB
- Well known in globular clusters, e.g. 47 Tuc (King et al. 1985)
- And also in LG dwarf galaxies (Gallart 1998)

Girardi et al. (2000):

Cassisi et al. 1999:
Thermally pulsing AGB

- double-shell phase, marked by thermal pulse cycle: He-shell flash $\rightarrow$ envelope expansion and H-shell extinction $\rightarrow$ He-shell extinction $\rightarrow$ envelope contraction and H-shell ignition $\rightarrow$ quiescent H-shell burning

$L$ evolution (Wagenhuber & Weiss 1994):

- rich nucleosynthesis (third dredge-up, hot-bottom burning, $s$-process) and mass loss
- rich photometry (late-M and C stars, pulse cycle, long-period and semiregular variables)
TP-AGB evolutionary sequences

- Boothroyd & Sackmann (1988abcd): excellent description of CMLR, pulse cycle, dredge-up, intershell properties
- Vassiliadis & Wood (1993): first extended set, use of empirical mass-loss formula \( \log \dot{M} \) with superwind phase:

\[
\frac{1}{M} \lesssim 2.5 \lesssim \frac{5}{M}
\]

- many others sets now available. Common features: detailed but few tracks, lack of 3rd dredge-up, use of modified Reimers \( \dot{M} \)
The third dredge-up(s)

- energy released by He-shell flash expands the envelope
- the H-burning shell is temporarily extinguished
- envelope convection can reach the C-rich material produced by the flash \( \rightarrow \) 3rd dredge-up
- after many 3rd dredge-up episodes, \((C/O)_{\text{surf}} > 1\) \( \rightarrow \) transition from M to C-star

This works only for more massive AGB models \((M \gtrsim 3 \, M_\odot)\) after a few thermal pulses.
The C-star mystery (Iben 1981 and later):

- Evolutionary models indicate a minimum core mass \( \gtrsim 0.65 M_\odot \) for dredge-up \( \rightarrow \text{then a minimum} \)
  \( M_{\text{bol}} \sim -5.5 \) for C-star models
- But Magellanic Cloud C-stars have \( M_{\text{bol}} \) between \(-3\) and \(-6\)!
- And C-stars in LMC clusters have TO-mass as low as \( M \sim 1.5 M_\odot \) (Frogel et al. 1990, Marigo et al. 1996)

The question is open. Possible solutions (yet to be fully tested):

Indications about the third dredge-up

- in models in which 3rd dredge-up is imposed and parameterized
- calibrations using the C-star luminosity function indicate:
  - minimum mass of C-stars is \( \sim 1.2 \, M_\odot \)
  - \( \lambda \sim 0.5 \) even for low masses
  - C/M ratio decreases with \( Z \)

(Marigo et al. 1999)
Carbon stars

- Russell (1934): a consequence of high binding energy of CO molecule, excess element makes
  - if $C/O < 1 \rightarrow$ TiO, VO, H$_2$O bands $\rightarrow$ M star
  - if $C/O > 1 \rightarrow$ C$_2$, CN bands $\rightarrow$ C star
- old C-star models used constant opacities, and were far too hot and had too high $C/O$ ratio
- Marigo et al. (2002–2013) solution: to consider the varying molecular opacities as $C/O$ increases
Hot-bottom burning

- in more massive AGB stars \((M > 3 \, M_\odot)\), bottom of convective envelope reaches 
  \(T_b \gtrsim 5 \times 10^7 - 10^8 \, \text{K}\) \(\rightarrow\) CN cycling in the envelope
- burns H into \(^4\text{He}\), \(^{12}\text{C}\) into \(^{13}\text{C}\) and \(^{14}\text{N}\) \(\rightarrow\) prevents C-star phase, yields primary \(^{14}\text{N}\) (Boothroyd et al. 1993)
- \(^7\text{Li}\)-rich luminous AGB stars (Smith & Lambert 1989) via Cameron-Fowler mechanism (Sackmann & Boothroyd 1992)
- these stars do not follow a CMLR but are above it (Blöcker & Schönberner 1991 and later) \(\rightarrow\) \(L_{\text{AGB}}^{\text{max}}\) limit is not determined by \(M_{\text{Ch}}\) but by mass-loss
- later conversion to C-star phase is possible, but only after mass-loss peels the envelope off \(\rightarrow\) obscured luminous C-stars (Frost & Lattanzio, van Loon et al.)

Marigo et al. (1998)
Blöcker & Schönberner (1991)
AGB termination

- If $\dot{M} = 0$, AGB termination would be given either by $M_{\text{env}} \rightarrow 0$ or $M_{\text{core}} \rightarrow M_{\text{Ch}}$
- Actually, interplay of mass-loss and dredge-up determine it
- Possible mass-loss formulas
  - Reimers (1975)-like: not justified, but commonly used → no superwind
  - Correlated with pulsations period: empirical (Vassiliadis & Wood 1993) and theoretical (Bowen & Willson 1998) → superwind-like
- Constraints: observed initial–final mass relation for white dwarfs, and $L_{\text{AGB}}^{\text{max}}$ for masses $M \lesssim 3 M_\odot$
- Note: in more massive AGB stars, HBB is on → $L_{\text{AGB}}^{\text{max}}$ is not limited by $M_{\text{Ch}}$ but by onset of superwind
End of lecture 1

Tomorrow: isochrones, stra clusters, binaries