Explaining the Galactic Interstellar Dust Grain Size Distribution Function

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ABSTRACT

We present here a new theoretical model designed to explain the interstellar dust grain size distribution function (IDGSDF), and compare its results with previous observationally derived distributions and with previous theoretical models. The range of grain sizes produced in the late stages of stars with different masses is considered, and folded into a model which takes into account the observed changes in the historical local star formation rate. Stars in different mass ranges reach their grain producing epochs at times whose mass dependence is quantifiable, and the range of grain sizes produced has also been estimated as a function of stellar mass. The results show an IDGSDF which has a global slope comparable to the observationally derived plot and three peaks at values of the grain radius comparable to those in the observationally derived distribution, which have their ultimate origin in three major peaks which have been observed in the SFR over the past 15 Gyr. The model uses grain-grain interactions to modify pre-existing size distributions at lower grain sizes, where collisions appear more important. The interactions include disruption by collisions as well as coagulation to form larger grains. The initial distributions are given a range of initial functions (flat, Gaussian, fractal) for their physical parameters, as well as geometrical forms ranging from spherical to highly elongated. The particles are constrained in an imaginary box, and laws of inelastic collisions are applied. Finally we combine the two models and produce an IDGSDF which is a notably good match to the observational fit, and specifically at small grain radii reproduces the data better than the "SFR model" alone.

Subject headings: ISM: dust
1. Introduction

Banard and Trumpler’s observations first confirmed the existence of obscuring clouds of interstellar dust (see Whittet 1992), showing that the wavelength dependence of the interstellar extinction is proportional to $\lambda^{-1}$, which implied submicron-size dust grains. Although its mass constitutes only some 1% of the mass of the ISM gas clouds the dust plays a very important role both as the major light blocker for observations and as a seed coagulator to form giant molecular gas clouds which give birth to the stars which in turn produce new grains (red giant winds, supernovae, novae) and also destroy some of them by sputtering, shattering and evaporation in fast interstellar shocks. But it is the chemical composition of the dust grains which has been the most difficult problem to address, since the available observational data do not really provide strong enough constraints. Several solids have been suggested: ices, silicates, various carbon compounds, metals, and also complex organic molecules. Graphite has been a popular grain constituent since it was associated with the 2175 extinction bump (Stecher & Donn 1965), and there are two widely observed features that characterize dust grains: silicate features in the infrared (Woolf & Ney 1969, Gillett & Forrest 1973). Interstellar dust grains play an important role in processes in the interstellar medium, such as the absorption of UV radiation, emission of infrared radiation and heating of the surrounding gas through the photoelectric effect, which leads to the emission of energetic electrons. Catalytic processes on grain surfaces give rise to molecular hydrogen (Gould & Salpeter 1963, Hollenbach, Werner & Salpeter 1971) and probably other molecules (Hasegawa, Herbst & Leung 1992). Dust grains also take part in the dynamics of the interstellar clouds by coupling the magnetic field to the gas in regions of low fractional ionization and by transferring stellar radiation pressure to the gas (Draine 2003). Dust grains consisting of oxides, silicates, carbonaceous matter, diamond, SiC, and other carbides form by vapor-phase condensation in the outflows of the late-stage stars (Roberge et al. 1995, Kwok 2002, Matsuura et al. 2004), in stellar winds (Ferrarotti & Gail 2002), and in the rapidly expanding gas shells of nova and supernova explosions (Dey et al. 1997, Starrfield et al. 1997, Pontefract & Rawlings 2004).

Stellar winds, jets, and supernovae shock waves tend to destroy grains (Seab & Shull 1983, Jones et al. 1997). However a large body of observational evidence indicates that the majority of grains forming in circumstellar outflows and residing in the ISM have undergone only slight destruction by subsequent interstellar processes (Jones et al. 1994, Freund & Freund 2006).

Most studies of grain size over the last half century have assumed a size distribution that is either a power law or a power law with exponential decay. These power laws have been used for two reasons. First, they provide a good fit to the observed wavelength dependence
of extinction with as few as two populations of grains. Second, formation and destruction processes such as shattering and coagulation that modify grain sizes along many sight lines tend to produce a power-law size distribution (Biermann and Harwit 1980, Hayakawa and Hayakawa 1988, O’Donnell and Mathis 1997). Modelling the observations began with Oort and van de Hulst (1946), who fitted extinction at visible wavelengths assuming ice grains and a size distribution resembling a power-law with negative slope. Greenberg (1968) expanded this work using spheres, cylinders, and spheroids, combined with many different materials and grain orientations. Mathis et al. (1977) constructed their classic interstellar dust model on the basis of the observed extinction of starlight for lines of sight passing through diffuse clouds, obtaining a power-law IDGSDF with slope -3.5.

Weingartner (2006) suggested that for suprathermal rotating grains the misalignment process with respect to the interstellar magnetic field may be stronger for carbonaceous dust than for silicate dust. This could result in more efficient alignment for silicate grains but poor alignment for carbonaceous grains, leading towards a possible explanation of their significantly different size distributions (see Weingartner and Draine 2001 and Fig. 1).

The present article is organized as follows: In Section 2 we present the distribution functions derived from the observations by several authors, in Section 3 we develop a model in which the production history of grains, originating in the late evolutionary stages of stars with a range of stellar masses, can be used to predict the IDGSDF, with considerable success, in section 4 we develop a model which accounts for the effects of grain-grain collisions on the IDGSDF, in section 5 we combine the collisional model with function due to stellar birthrate, and find that the agreement with observations is improved, and in section 6 we present our conclusions.

2. The curves to fit

We have taken an average over the free parameters of the observationally derived size distributions for carbonaceous and silicate grain populations in different regions of the Milky Way, the LMC, and the SMC by Weingartner and Draine (2001) which we show in Fig.1. The size distributions include sufficient very small carbonaceous grains (including polycyclic aromatic hydrocarbon molecules) to account for the observed infrared and microwave emission from the diffuse interstellar medium. Their distributions reproduce the observed extinction of starlight, which varies depending on the interstellar environment through which the light travels. These variations can be roughly parameterized by the ratio of visual extinction to reddening. They adopt a fairly simple functional form for the size distribution, characterized by several parameters. They tabulate these parameters for various combinations of values
for the visual extinction reddening and the C abundance in very small grains. They also find size distributions for the line of sight to HD 210121 and for sight lines in the LMC and SMC. For several size distributions, they evaluate the albedo and scattering asymmetry parameter and present model extinction curves extending beyond the Lyman limit.

We also include for comparison the Kim et al. (1994) observationally derived distributions for the Galaxy using the maximum entropy method (MEM) for two cases: diffuse ISM ($R_V=3.1$) and a dense cloud region ($R_V=5.3$). And also we take the averaged Clayton et al. (2003) observationally derived distributions using MEM again for the Galaxy and also for the Magellanic Clouds.

In Fig. 1a we show the comparison among observationally derived (Weingartner and Draine 2001) size distribution functions for carbonaceous dust grains: in the Galactic ISM (full line), in the LMC (dotted line), and in the SMC (dashed line). One can see the quite similar global behaviour, especially for large grains, and the main difference is the absence of the two peaks for small grains in the SMC. In Fig. 1b we show the same as in the previous figure, but for silicate dust grains. Here the three distribution functions are very similar.

The Magellanic clouds have low metallicities in comparison with the Galactic ISM: the SMC even lower than the LMC. The observations show some differences in the dust size distribution functions: although the LMC has a very similar distribution of sizes to the ISM, the SMC shows only one peak for both carbonaceous and silicate grains. Reach et al. (2000) point out that SMC extinction curve measurements are biased toward hot, luminous stars so that very small grains may have been destroyed along these sight lines. But another possibility, in the light of the present work, is that the very low metallicity of SMC (-0.65 dex (Russell and Bessell 1989)) could greatly decrease the probability of collisions among dust grains, so that the smallest grains are much less numerous than in the LMC, and a fortiori than in the Galactic ISM. Our modelling process is designed principally to fit the Galactic ISM data, since we are in a better position to give valid input parameters to the models. The LMC and SMC data are included for a general comparison.

We can see how, in both Figs. 1a and 1b, the curves by Kim et al. (1994) and by Clayton et al. (2003), show the two peaks for carbonaceous particles near -2 and -1 in logarithmic microns, in agreement with Weingartner and Draine (2001).

3. The model of grain production and survival as a function of epoch

As the majority of ISM dust grains are assumed to be formed in the circumstellar zones of late stage stars (mainly red giant stars, asymptotic giant branch stars and supernovae)
and since those stars present during the last 10 Myr (the lifetime of grains) are those of an initial mass range but evolved to the present epoch, the range of ages of these stars are between 13.75 Gyr and 0 Gyr with the most massive also being the youngest and with the highest metallicity. Then we can expect a very different range of physical conditions for grains produced in older stars (low mass) from those for grains produced in younger stars (high mass). Moreover, the effective destruction of grains in supernova remnants affects only the grains of lower sizes, which gives an explanation for the correlation between observed major peak in radius (to the right in the figures below) and the late-stage stars arising from late evolutionary stages of younger more massive stars. As one goes to the left in the figures, to smaller grain radii, the production of grains is from older, less massive, stars which have evolved to the red giant stage. We note here the observation of three prominent peaks in the star formation rate in the local Galaxy (Rocha-Pinto et al. 2000) and also three peaks in the IDGSDF of carbonaceous grains. We will show that there is a causal link between these three star forming periods and the 3 peaks observed for the sizes of the grains. For silicates the observed IDGSDF shows only one peak quite close in grain radius to the major peak observed for carbonaceous grains. The observed peak in IDGSDF for small grain radii can be due to those grains produced during the last 100 Myr around the late stages of low mass stars (close to 1M⊙ from the first major peak in the SFR). This is because there the velocities of grains in the expanding shells are lower than those in stars with higher masses so that the time for possible collisions among grains is greater leading to a higher probability of shattering and smaller sizes. In the second peak we have grains produced by stars of intermediate age, (i.e. of intermediate mass from the second peak in the SFR) where the velocities of grains are intermediate, so that some bigger grains can escape from the high density shells where the shattering would otherwise break them into smaller grains. The last peak with the largest grains can come from SNe where the expansion velocities are so high that the biggest grains can escape from the shells into the ISM where the densities are so low that the relatively few collisions occur. Nozawa et al. (2007) have found that for Type II SNRs expanding into the ISM with a number density of 1cm⁻³, small grains (≤0.05 µm) are completely destroyed by sputtering in the postshock flow, while grains with 0.05-0.2 µm are trapped in the dense shell behind the forward shock, and very large grains (≥0.2µm) are ejected into the ISM without decreasing their sizes significantly. So, we compute the final size distribution of dust grains using the expression:

\[ N(r) \propto SFR(T) \left( \frac{15}{T} \right)^{-2.35} mR(m)r^4 r^{-3} \]  

(1)

where \( N(r) \) is the number of grains with size \( r \), \( SFR(T) \) the star formation rate taken from Rocha-Pinto et al. (2000) as a function of epoch, the second term is the fraction of stars that due to their evolution are now in their final evolutionary stages. For this we use the
Salpeter mass function and a functional form to relate masses and ages approximated by $\text{age} \propto \frac{15}{m^2}$ with age in Gyr and $m$ in solar masses, $m$ represents each stellar mass, $R(m)$ is the fraction of mass expelled into the ISM by a star with a given mass $m$ in its last evolutionary stage (Nieuwenhuijzen and Jager 1990, Holzwarth and Jardine 2007, Benaglia et al 2001), and the $r^{-3}$ term is included to ensure the division by the mass of each grain of radius $r$, and finally we multiply by a factor $r^4$ to yield graphs which are easy to inspect and reflect the mass distribution. The result is the full line of Fig.4 a. We can see that this scenario accounts rather well for important parameters of the data, notably the overall slope of the distribution for both carbonaceous and silicate grains, and the presence of three local maxima in the distribution. However there is an increasing trend to disagreement between the model and the observations at small values of grain radius. For this reason we explored how introducing grain-grain collisions into the model affects the predicted distribution.

4. The pure collisional model

In Casuso and Beckman (2002) we developed an analytical approximation to a physically parametrized treatment of the observed mass functions (MF’s) of gas clouds in the ISM, and in Casuso and Beckman (2007) we presented a simple numerical approach to the gas cloud mass distribution function by simulating formation and destruction of gas clouds and gas clumps in the Galactic ISM. Here we have adapted the same basic numerical approach to describe the observed distribution by size of interstellar dust grains both for silicates and for carbonaceous grains. The idea is to parametrize the size distribution function of dust grains at a given epoch as a function of, essentially, their initial distributions of density, temperature, velocity and mass. The position and velocity distributions are three-dimensional. We assume different sets of initial (i.e. at $t = 0$) distribution functions, (Gaussian, flat, power-law) for temperatures, densities, masses and velocities of an initial sample of $\sim 1000$ dust grains, either pressure bound or gravitationally bound, evolving within a confinement volume specified as a cubic box, and arranged initially as knots in a cubic grid. Each grain has an effective volume which depends on its mass and density, via $V \sim M/\rho$ if a grain is taken to be spherical, or via $V \sim M \beta^2/\rho$ if the grain is taken to be elongated, with $\beta$ being the ratio between the longest axis and the other two (see Casuso & Beckman 2002). After a given evolution time, with a given initial set of velocities, we consider that two of the grains have collided if the distance between their centres is less than or equal to the sum of their radii $r$, which are calculated using $r = \left(\frac{3M}{4\pi \rho}\right)^{1/3}$. To take particular values for the main physical parameters (density, temperature, velocity, and mass) for the different distribution functions, we take the expressions already explained in Casuso and Beckman (2007).
At each time step if the temperatures of the two colliding grains are greater than or less than the adopted critical temperature, and the relative velocity of the grains is less than the critical velocity, and the densities of the two grains are less than the critical density, i.e., the collision is completely inelastic, then we assume that the grains merge to yield a single grain with mass equal to the sum of the two input masses. If all the previous conditions hold but the relative velocity between grains is greater than the critical velocity, then we assume both grains disappear by evaporation into the ISM. If the temperatures of the two grains are less than the critical temperature, the two densities are greater than the critical density, and the relative velocity is greater than the critical velocity, we assume that each of the two grains breaks into two subgrains, for simplicity each subgrain having half of the mass of the parent grain. If the temperature of one grain is less than the critical temperature, the temperature of the other is greater than the critical temperature, and the density of the first grain is greater than the critical density, while that of the second grain is less than the critical density, then the first grain remains unchanged while the second grain breaks into two subgrains each with half the mass of the parent grain.

All the kinematics are computed subject to momentum conservation in each kind of collision (see Casuso and Beckman 2007).

We also have followed Weingartner and Draine (1999) in taking a lifetime for dust grains close to $10^7$ yr. They found that the observed elemental depletions in the interstellar medium could be caused by accretion onto grains if the timescales for matter to cycle between interstellar phases are close to $10^7$ yr.

For our first collisional numerical model we use fractal initial distribution functions for all parameters, i.e., assuming a distribution function following a power-law for each parameter divided by its width and both powered to -3.35. For all parameters (width, and critical value) we use for: temperatures (80 K, 90 K), densities (2 $g/cm^{-3}$, 2 $g/cm^{-3}$), masses (5$x10^{-21}$g, none), velocities ($1x10^7cm/s^{-1}$, $1x10^6cm/s^{-1}$). We can see in fig 2 how the output does give fair agreement with the observations over a selected range of small grain radii.

In order to simplify the computation and then to minimize the errors we have renormalized the distances among grains, and then coherently the times and velocities involved. To do this, we have reduced the distances and velocities among grains by a factor of $10^{-13}$, then we had that 1 Myr is one step of our modelling. In fact, it is the same that to take a run of $10^7$ steps of 1 second each taking real distances among the grains (near $10^7$ cm) and velocities of near $10^6$ cm.s$^{-1}$. Then the numbers taken for computation are: mean distances among grains: near $2x10^{-6}$ (normalized units), average relative velocity: near $10^{-6}$ (normalized units) and time step: 1 Myr.
The range of temperatures and densities which give at least partial fits for the interstellar dust grains agree with those of Horn et al. (2008) who developed a model which simulated stochastic heating and radiative cooling, taking into account the discrete nature of the UV photons of the radiation field.

Within the range of parameters used here the modification when we use elongated dust grains in which one dimension is up to 10 times bigger than the other two is not appreciable for the range of sizes of interest.

Although the main goal of the present work is to explain the three peaks obtained for the observationally derived IDGSDF by our model of grain production and survival as a function of epoch, we need collisions among grains to explain the IDGSDF below -2 in log sizes. To do this we vary three initial distribution functions for the main parameters (temperatures, densities of grains, masses of grains and relative velocities): power-law, flat and Gaussian. Our best fit for this range of lower sizes is for the power-law distribution as we can see in Fig. 2. Nevertheless, to show the limitations of the model we have varied all the parameters and models. In Fig. 3a we can see how the variation of densities or masses of grains by one order of magnitude result in significant disagreement with the best fit using power-law initial distributions. In Fig. 3b we see again a significant variation in the results when one change the slope of the power-law by an order of magnitude. But when we change the values of temperatures, velocities, grain elongations or initial relative distances among grains, the final result is not significantly changed at this range of sizes. The same is obtained when we run the model changing the age, even for times longer than the expected lifetime of most grains (near 10 Myr), with no significant changes even for times as long as 100 Myr. In Fig. 3c we show the results of flat initial distributions varying densities or masses of grains by an order of magnitude. Although the changes in the results are significative the main conclusion is that the flat distribution does not fit the observations at the range of sizes of our interest. In Fig. 3d we have changed the initial relative distances among grains by a factor of 10, and the results does not change significantly and again are not in the range of sizes of interest. In Fig. 3e we have evolved the grains until ages of 100 Myr and the result does not change in the direction of best fit at the range of sizes of our interest. The results obtained using initial Gaussian distribution and ages of 100 Myr are very similar to those obtained in Fig. 3e. For variations of an order of magnitude of temperatures, velocities and grain elongations using flat initial distributions, the results are not changed significantly. In Fig. 3f we show the results of the only parameters whose changes lead to significantly variations in the output curves taking Gaussian initial distributions: the densities or masses of grains changed by a factor of 10.

Weingartner (2006) has found that for suprathermal rotating grains the misalignment
process with respect to the interstellar magnetic field, may be stronger for carbonaceous
dust than for silicate dust. This could result in efficient alignment for silicate grains but
poor alignment for carbonaceous grains, and perhaps offers an explanation of the significant
differences between the two size distributions (see Weingartner and Draine 2001 and Fig. 1)
This is because the greater rigidity of the motion for silicates gives them greater protection
against the high velocity collisions which leads to a higher probability of larger grain sizes
leading to a single peak in the IDGSDF, in contrast with the three peaks for the carbonaceous
grains. Moreover, in the present work we find that the two peaks observed for carbonaceous
grains at small radii may be better explained assuming an initial distribution for parameters
following a power-law function, i.e. showing fractal behaviour (scale independence), which
is characteristic of distributions due to supersonic turbulence (Larson 1981, Falgarone et al.

5. The complete model

As the final stage we apply the collisional model to our model of grain production ages.
Since collisions are not important for larger grain sizes as we explained in Section 3, we
focus our attention on the modifications of the IDGSDF through collisions at lower sizes,
the only result that yields a peak (in fact two) near -3 on our scale of log (microns). For
the simple collisional numerical model we opted to use the case of spherical dust grains, and
fractal initial distribution functions for all parameters, i.e. assuming a distribution function
following a power-law for each parameter divided by its width, and both with an exponent
-3.35. For all parameters (width, and critical value) we use for: temperatures (80 K, 90
K), densities (2 g×cm$^{-3}$, 2 g×cm$^{-3}$), masses (5×10$^{-21}$g, none), velocities (1×10$^7$cm×s$^{-1}$,
1×10$^6$cm×s$^{-1}$). observed. The result, shown in Fig.4b shows improved agreement with the
observationally derived distribution of Weingartner and Draine (2001), especially at small
value of grain radius.

When we compare with previous theoretical models we can see in Fig. 5 how the main
peak at near -1 (log size) is well predicted by O’Donnell and Mathis (1997) and also in the
classic paper MRN (1977), but our new model also fits the smallest range of sizes, i.e. below
-2, explaining not only the main peak but also the other two peaks.
6. Conclusions

Knowing that ISM dust grains are generally formed in the outer atmospheres of stars in the late stages of their evolution (mainly red giants, asymptotic giant branch stars and supernovae) and that grain lifetimes can be estimated as less than 100 Myr (in fact ∼10 Myr), we have taken an observational version of the local stellar birthrate to calculate the radius distribution of grains surviving today. To do so we computed the mass distributions of stars present over this period assuming a standard IMF, the observed stellar birthrate function, and stellar lifetimes as a function of their masses, and folded in the initial distribution of dust grain sizes at each epoch, according to a simple prescription of grains produced in different stellar mass ranges. The differences between the size distribution of grains produced in the older, low mass stellar population and those produced in younger, high mass stars include the tendency of the smaller grains to be preferentially destroyed by shocks in the post-SN environment. Under these assumptions the major peak in the grain size distribution comes from dust produced in the younger more massive stars, and the three peaks in the IDGSDF of carbonaceous grains correspond to the three maxima in the local SFR, as detailed in Rocha-Pinto et al. (2000). This correlation between the three main star formation events in the Galaxy and the three peaks observed for the sizes of grains appears clear. For silicates the observed IDGSDF shows only one peak, close to the biggest peak in the distribution observed for carbonaceous grains. This increases the plausibility of the scenario in which the peak for the carbonaceous grains at the smallest radii is associated with those grains produced during the last 10 Myr near the late stages stars of lower masses (near 1M⊙ arising from the earliest major peak in the SFR) because in that case the velocities of grains in the expanding shells are lower than in stars with higher masses so that the time interval for collisions among grains is greater so the probability of shattering is higher and the mean sizes of the grains are lower. For the second peak we have grains coming from stars of intermediate age (intermediate mass coming from the second peak in the SFR) where the velocities of grains are intermediate and so some of the larger grains can escape from the high density shells where the shattering can break them into smaller grains. Finally the third peak, with the largest grains, can come from SNe where the expansion velocities are so high that the largest grains can escape from the shells into the ISM where the densities are so low that the collisions occur relatively infrequently. The observed difference in the slope of the distribution function for carbonaceous (close to 0) and silicate grains (close to 1) may be due to the mechanism proposed by Weingartner (2006) for suprathermal grains, whereby the misalignment with respect to the interstellar magnetic field, may be stronger for carbonaceous dust than for silicate dust. This would lead to greater collisional effects for the carbonaceous grains, changing the initial slope to that observed. The global features of the IDGSDF are rather well reproduced by the model in which we use only the dust
sizes yielded by the stellar production process and the accompanying modifications due to shattering in the local environment of the late stages of the stars, together with a stellar birth rate function from observations. However a closer fit to the carbonaceous grain distribution at the low radius end is obtained if we also include the modelling of longer term grain-grain collisions processes in the ISM.

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Figure Captions

Fig. 1.— a) Comparison among observationally derived size distribution functions for carbonaceous dust grains: Weingartner and Draine (2001) ISM (full line), Weingartner and Draine (2001) LMC (dotted line), Weingartner and Draine (2001) SMC (dashed line), Kim et al. diffuse ISM (1994) (long dashed line), Kim et al. dense ISM (1994) (dotted-dashed line), and Clayton et al. averaged ISM+Magellanic clouds (2003) (dotted-long dashed line). One can see the very similar behaviour of the results from different authors for large grains, and also for small grains (with the exception of the SMC). b) The same as in the previous figure, but for silicate dust grains. Here all distribution functions are very similar.

Fig. 2.— Present day size distribution function of carbonaceous-silicate dust grains in the ISM of our Galaxy. Data are the same shown in Fig 1 for the ISM. For our collisional numerical model (full line) we are only interested in the lower sizes. We assume fractal initial distribution functions for all parameters, i.e. assuming a distribution function following a power-law for each parameter divided by its width and both powered to -3.35. For all
parameters (width and critical value) we use for: temperatures (80 K, 90 K), densities (2 $g/cm^3$, 2 $g/cm^3$), masses ($5 \times 10^{-21}$ g, none), velocities ($1 \times 10^7 cm/s$, $1 \times 10^6 cm/s$).

Fig. 3.— a) The same as in figure 2 (full line), but varying several parameters to test the sensitivity of the model: density multiplied by 10 or average masses of grains multiplied by 0.1 (dashed line), density multiplied by 0.1 or average masses multiplied by 10 (dotted line). b) The same as in Fig. 1 but varying the slope of the fractal initial distribution: -2.35 (full line), -4.35 (dotted-dashed line). c) The values of Fig. 1 but taking flat initial distributions for all parameters: density multiplied by 10 or masses multiplied by 0.1 (full line), density multiplied by 0.1 or masses multiplied by 10 (dotted line). One can see that the fit is not good for the range of sizes of interest. d) The same as in c) but varying the initial relative distance among grains by a factor: 10 (full line) and 0.1 (dotted line). e) The same as in c) but varying the age: 1 Myr (full line), 100 Myr (dotted line). Note that a very similar variation is obtained for a Gaussian initial distribution for these ages. f) Here we have taken a Gaussian initial distribution of values of parameters, for all parameters (width of gaussian, center of gaussian, and critical value) we use for: temperatures (80 K, 80 K, 90 K), densities (2 $g/cm^3$, 2 $g/cm^3$, 2 $g/cm^3$), masses ($5 \times 10^{-21}$ g, $2 \times 10^{-15}$ g, none), velocities ($1 \times 10^7 cm/s$, $1 \times 10^6 cm/s$, $1 \times 10^6 cm/s$). But we have varied: density multiplied by 10 or average masses multiplied by 0.1 (full line), density multiplied by 0.1 or masses multiplied by 10 (dotted line).

Fig. 4.— a) The theoretical model taking into account the dust production in stars with different ages originating from three peaks in the Galactic SFR as a function of epoch (full line), data as in Fig. 1. b) Model produced by folding the stellar grains production curve with the effects of production and destruction in the circumstellar shells by collisions from Fig. 2 (full line).

Fig. 5.— Comparison with previous models: The full line is our best fit model, the dashed line shows the O’Donnell and Mathis (1997) model predictions, the dotted line shows the “classic” MRN (1977) model predictions. One can see that the three fits agree quite well within the differences among observationally derived curves in the range of grain sizes larger than -2 in log (microns), but our new model fits also the smallest range of sizes, i.e. below -2.